# THE FREE TRADE IMPLICATIONS OF DIFFERENTIAL FERTILITY AND LONGEVITY BETWEEN COUNTRIES: A 3x2x2 OVERLAPPING-GENERATIONS GENERAL EQUILIBRIUM ANALYSIS

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CEMAL EREN ARBATLI

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To My Family and Ekim

I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Arts in Economics.

---------------------------------------------
Assoc. Prof. Serdar Sayan
Supervisor

I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Arts in Economics.

---------------------------------------------
Asst. Prof. Tarık Kara
Examining Committee Member

I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Arts in Economics.

----------------------------------------------
Asst. Prof. Levent Akdeniz
Examining Committee Member

Approval of the Institute of Economics and Social Sciences

---------------------------------
Prof. Erdal Erel
Director

# ABSTRACT

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Arbatlı, Cemal Eren

M.A. in Economics

Supervisor: Assoc. Prof. Serdar Sayan

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This study employs two variants of a three-generation, two-sector and two-factor overlapping-generations model with lifetime uncertainty and altruistic agents to investigate the welfare implications of fertility and longevity differentials between trading countries. In the first model (the basic model), agents optimistically assume that they will survive into old age with certainty and have a constant saving rate. Agents in the second model (the anticipation model), on the other hand, correctly anticipate the mortality rates and adjust their saving rates accordingly. The models are solved both under autarky and a Heckscher-Ohlin free trade setting. The analytical results on the direction of the effects of changing population dynamics under lifetime certainty are complemented by numeric results under lifetime uncertainty that provide important insights on how increasing longevity and declining fertility rates may affect the welfare prospects of countries that are at different stages of demographic transition. The solutions reveal that longevity differentials are sufficient to create grounds for trade between countries with equal as well as different population sizes. In a closed economy, when mortality rates are perfectly anticipated, the marginal benefit of higher longevity gradually diminishes as the life expectancy increases and/or the fertility rate decreases. Consistently with this result, as demographic transition proceeds, free trade under longevity differentials tends to make the low-mortality country better off and the high-mortality country worse off compared to autarky.

*Keywords:* Dynamic trade, Life expectancy, Fertility, Lifetime uncertainty, Overlapping-generations general equilibrium model, Heckscher-Ohlin.

# ÖZET

## TİCARET YAPAN ÜLKELERİN DOĞURGANLIK ORANLARI VE YAŞAM UZUNLUKLARI ARASINDAKİ FARKLILIKLARIN SONUÇLARI: BİR 3x2x2 ÇAKIŞAN-NESİLLER GENEL DENGE ANALİZİ

Arbatlı, Cemal Eren

İktisat, Master

Tez Yöneticisi: Doç. Dr. Serdar Sayan

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Bu çalışma, üç-nesil, iki-sektör ve iki-faktörlü, içinde çocuklarının tüketimini artırma yönünde fedakarlık yapmaya hazır bireyler ve yaşam süresi hakkında belirsizlik barındıran bir çakışan nesiller modelinin iki farklı versiyonu çerçevesinde, ticaret yapan ülkelerin doğurganlık oranları ve yaşam uzunlukları arasındaki farkların bu ülkelerin refah seviyelerini nasıl etkilediğini incelemektedir. İlk modelde (basit model) bireylerin yaşam süreleri hakkında aşırı iyimser oldukları ve kesinlikle yaşlılık dönemlerini görecekmiş gibi davrandıkları ve dolayısıyla tasarruf oranlarının sabit kaldığı varsayılmaktadır. İkinci modelde (doğru önsezi modeli) ise bireylerin yaşlılığa geçemeden ölme olasılıklarını kesin olarak bildikleri ve tasarruf oranlarını bu olasılığa göre belirledikleri varsayılmaktadır. Bu çalışmada, her iki model hem kapalı ekonomi, hem de Heckscher-Ohlin tarzı serbest ticaret rejimi altında çözülmektedir. Yaşam süresi hakkında belirsizlik yokken elde edilen ve değişen nüfus dinamiklerinin ekonomiyi hangi yönde etkilediğine dair analitik sonuçlar, yaşam süresinde belirsizlik varken elde edilen nümerik sonuçlarla desteklenmiştir. Bu sonuçlar, artmakta olan yaşam süresinin ve düşmekte olan doğurganlık oranlarının demografik dönüşümün farklı evrelerindeki ülkelerin ekonomik refahı açısından ne gibi sonuçlar doğurabileceğine dair önemli ipuçları sağlamaktadır. Çalışmada sadece yaşam süresi farklarının bile iki ülke arasında ticaret olabilmesi için yeterli olduğunu gösterilmektedir. Kapalı ekonomi rejimi altında, eğer ölüm oranları bireylerin tarafından net olarak biliniyorsa, daha uzun yaşam süresine sahip olmanın marjinal getirisi beklenen yaşam süresi arttıkça ve/veya doğurganlık oranları düştükçe kaybolmaktadır. Bu sonuçlarla uyumlu olarak, demografik dönüşüm ilerdikçe serbest ticaret, daha yüksek ölüm oranına sahip ülkeyi kapalı ekonomi durumuna kıyasla olumsuz, daha düşük ölüm oranına sahip ülkeyi ise olumlu etkileme eğilimindedir.



*Anahtar Sözcükler:* Uluslararası ticaretin dinamik dengesi, Ortalama yaşam beklentisi, Doğurganlık, Yaşam süresinde belirsizlik, Çakışan-nesiller genel denge modeli, Heckscher-Ohlin.

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# CHAPTER 1:

# Introduction

While population growth rate is a compact demographic indicator frequently used in economic analysis, age structure of population is rarely taken into account by economists. As argued by Bloom, Canning and Sevilla (2001), the economic significance of age structure of a population has been mostly overlooked by macroeconomists who have usually concentrated only on population growth as a determinant of economic growth. The authors rightly point out that relative sizes of different age groups in a population highly matter, because agents from different age groups with different objectives and capabilities behave differently. Based on the life-cycle hypothesis of Modigliani and Brumberg (1954), scholars recognized that accounting for the heterogeneity of agents across different age cohorts is very crucial in understanding the effects of demographic change. OLG models attributed to Samuelson (1957) and Diamond (1965) are considered as convenient analytical tools for this purpose, and they have been increasingly popular both in policy oriented and theoretical analyses, especially after the 1980s.

This thesis utilizes different versions of a perfect foresight[1] three-generation, two-sector and two-factor overlapping generations general equilibrium (OLG GE) model to investigate the individual roles of fertility and mortality rates in influencing the growth prospects of countries under autarky and free trade. Unlike most studies that examine the economic implications of changes in population growth rate as a compact demographic indicator, this study disaggregates population growth rate into mortality and fertility components.[2] Fertility and adult mortality rates enter these models as two exogenous and independent demographic parameters that affect saving and consumption behavior of economic agents in different ways.

The primary goal of the thesis is to examine the cross-border implications of differential population dynamics by analyzing the role of free trade in transmitting the effects of demographic shocks from one country to the other. As discussed in the next chapter, countries that are at different stages of development are usually at different stages of their demographic transitions, with visibly different population dynamics. Population aging in industrial countries has already started to be an important concern for policy makers. As for relatively younger developing countries, the demographic window of opportunity is still open thanks to the large share of working force in total population and relatively lower old age dependency ratio in these countries. However, these countries are not immune against economic spillover effects emanating from the rest of the world. Increasing flows of goods and services between industrial and developing countries due to intensifying economic globalization may raise the

[1] Here, perfect foresight means that agents can foresee the whole sequence of relative commodity and factor prices that will result from their decisions.
[2] Since the impact of migration is out of the scope of this thesis, immigration and emigration are assumed away in the models.

significance of trade induced spillover effects of differential population dynamics for growth and development.

Sayan (2005) is one of the pioneering studies that investigate the open economy implications of population dynamics. It provides a new explanation for why, in a dynamic OLG setting, trade may lead to deterioration in welfare for some of the trading parties. This explanation relies on the observation that differential population growth rates between trading countries may cause a divergence in relative factor abundances between countries creating a basis for trade. Using a Heckscher-Ohlin (HO) framework, Sayan (2005) establishes that free trade may be Pareto-inferior to autarky by lowering the welfare level of low population growth country without altering it in the high-population growth country in the long-run. The main motivation of this thesis is to extend this paper in two directions. The first one is the introduction of lifetime uncertainty through incorporation of adult mortality. Inclusion of lifetime uncertainty makes it possible to conduct demographic experiments where old age dependency ratio is not completely determined by fertility rates and can be freely adjusted by changing mortality rates. In addition to fertility rate differentials considered as the sole reason for diverging labor endowments between countries in Sayan (2005), this thesis allows differences in adult mortality to create differences in capital endowments between countries without necessarily changing the relative population sizes in these countries. Therefore, the introduction of adult mortality component as an additional factor influencing relative factor endowments in each country enriches the analysis of free trade under demographic change. The second extension to Sayan (2005) is the introduction of children as a third generation that totally depends on the adult generation for their livelihood. This extension amplifies the economic cost of high fertility by capturing the negative scale effect of increases in the youth dependency ratio. As also differently from Sayan (2005), the model in this thesis treats each generation as consumers of only one of the two goods instead of both goods. The relatively labor-intensive good is consumed only by children, whereas the capital-intensive good is only consumed both by adults and the elderly besides serving as an investment good at the same time. This asymmetry in consumer preferences across the two goods is shown to also play a role in the determination of relative prices.

Since the model in this thesis includes children as a third generation living concurrently with working adults and retired elderly at any period in time, it is capable of capturing the economic impact of changes in child dependency ratio, as well as changes in the old-age dependency ratio. This aspect of the model serves to highlight the welfare-deteriorating effect of increasing youth dependency ratio (Kelley and Schmitt, 1994), which is empirically identified to play a greater role in the economic slow down in developing countries than elderly dependency ratio (Bloom and Williamson, 1998). The two-generation OLG models, where agents enter the labor force immediately after birth, underestimate the negative impact of increasing fertility rates on the consumption of children and welfare of their altruistic parents. Departing from Bryant et al. (2003), this model includes altruistic child support so that changes in youth dependency ratio magnify the macroeconomic responses. In the first period, each agent spends her childhood as a dependent on her working parents who derive utility from the total amount of consumption allocated to their children. This formulation is similar to the bequest-as-consumption models where parents receive direct utility from the act of giving which Andreoni (1990) and

Michel (2004) refer as "warm glow giving" and "paternalistic altruism" respectively. The main difference between our framework and "paternalistic altruism" is that, in the models developed in this thesis, children do not carry over any benefits from their first period consumption to the next period when they become adults. Here, child consumption only provides a satisfaction for parents and does not accumulate in the form of human capital in the working age.

Besides the altruistic child support component described in the previous paragraph, the models in the thesis incorporates a bequest system due to the introduction of adult mortality rate that determines the share of adults who survive into old age. [3] In other words, agents face an exogenously given probability of death at the end of the second period[4] and those who cannot survive leave "accidental" or "unintended" bequests like in Hubbard and Judd (1987) and Pecchenino and Utendorf (1999). This formulation aims to capture the economic impact of intergenerational wealth transfers due to changing life expectancy via, in addition to the scale effect emanating from changing old age dependency ratio. Moreover, accounting for lifetime uncertainty is necessary in life-cycle models such as this one, because, contrary to what most OLG models without lifetime uncertainty assume, individuals often die with positive net assets which are transferred to younger generations. Kuehlwein (1993) finds strong empirical support for such a bequest motive. Furthermore, changes in the amount of bequests due to changes in longevity may have

[3] Following Pecchenino and Utendorf (1999), it is assumed that decreasing adult mortality causes an increase in the length of retirement rather than in the length of working period. This assumption is consistent with Hamermesh (1984) who finds that longer-lived workers do not significantly work longer than their shorter-lived colleagues.

[4] Infant mortality is not explicitly included as a separate population parameter because it can be easily incorporated into the model by adjusting fertility rates net of infant deaths.

considerable consequences for inter-temporal wealth allocation as well as wealth accumulation. In support of this argument, Gale and Scholz (1994) find that bequest motives are important determinants of savings behavior. Similarly, Kotlikoff and Summers (1981) argue that bequests constitute a larger fraction of national wealth than the life-cycle savings.

Concerning the life expectancy, the thesis compares the implications of two extreme assumptions with regard to the foresight of agents over their likelihood of death by analyzing two different models under autarky and trade. The first one, referred to as the "basic model", adopts the assumption that agents cannot anticipate the probability of death they face correctly and optimistically assume that they will survive into old age with certainty. As a result of this assumption, saving rates remain constant although mortality rate changes. In the second model, agents have perfect foresight over this probability and adjust their saving rates accordingly. The latter model, shortly referred to as the "anticipation model" throughout the thesis constitutes the other extreme. It is more compatible with the perfect foresight assumption over the factor and commodity prices and more realistic than the former model in the sense that in this model marginal propensity to consume out of total wealth is related to the probability of death and the rate of time preference like in Masson and Tryon (1990). Lee et al. (2001) also support this formulation by drawing on the role of mortality declines in the historical upward movement of saving rates. However, the extent of the impacts of demographic change and the age structure of a population on saving rates is open to debate. For instance, Deaton and Paxson (2000) find a very limited role for demographic change in altering the saving rates after analyzing the case of Taiwan. Therefore it is also useful to first look at the implications of the basic model with constant saving rates not only as a benchmark of analysis but also because of the standing debate over the extent of the effect of demographic change on saving rates. The thesis also discusses the qualitative difference between the responses of the two models to mortality shocks under autarky and trade. This difference underlines the importance of the debate on the relationship between demographic change and saving rates for predicting the economic consequences of population dynamics.

Population dynamics influence economic performance of a country by altering the age distribution not only in the long-run, but also in the short-to-medium run. Based on this observation, the demographic experiments conducted in the thesis are designed in such a way to capture the short-to-medium run as well as long-run effects of demographic change. Yet, the models in the thesis are not intended to serve for policy evaluations in real life. They are rather theoretical constructs to understand the way fertility and mortality dynamics and the resulting dependency ratios influence prices, capital accumulation, sectoral outputs and consumption. Hence, some of the demographic scenarios analyzed in the rest of the thesis only aim to see the responses of economic variables to demographic shocks and may not be consistent with real demographic trends.

The thesis is organized as follows: Chapter 2 gives an overview of past and projected demographic trends in different regions of the world and reviews the economic literature on demographic change with its cross-border implications. Chapter 3 introduces the basic model where agents do not correctly foresee adult mortality rates and presents analytical results under complete survival into old age. Chapter 4 discusses the results obtained from the numerical solution of the basic model under different demographic scenarios, including positive adult mortality. Chapter 5 first introduces the anticipation model where mortality rates are perfectly anticipated by agents and looks at the analytical results under complete survival to old age. Then, it analyzes various demographic scenarios under positive adult mortality. Chapter 6 introduces the free-trade versions of the two models and investigates the open economy implications of differential mortality and fertility dynamics between two trading countries as well as the nature of demographic shock transmission from one country to the other. Finally, Chapter 7 concludes the thesis.

# CHAPTER 2:

# Review of the Literature

The major decisions and experiences in our lives have profound demographic consequences in aggregate that in turn affect our welfare on various fronts. Besides one's life span, which is more or less exogenous to one's will, there are endogenous factors such as birth and marriage decision, choice of occupation, place of residence and retirement decision that have an impact on current and future economic outcomes. Obviously, these decisions are not made in a vacuum. Not only social and cultural, but also economic considerations play an important role in shaping these decisions. Therefore, it is a crucial task for economists to explore the underlying interplay between population dynamics and economic decisions. Similarly, it is of utmost importance not only to be able to predict future demographic trends but also to assess the impacts of a demographic change upon economies to overcome potential challenges demography poses.

When we look at the historical evolution of mortality and fertility rates, we see that before the twentieth century both fertility and mortality rates were high. Around

1800, starting in the northwestern Europe, mortality rates entered a dramatic downward trend. Outside Europe, mostly in countries that were called less developed, this decline began with a delay and only in the early twentieth century, and intensified after the World War II. This decline affected different age cohorts asymmetrically, mostly reducing infant and child mortality. On the other hand, fertility rates remained high well after mortality decline started, and this led to an increase in the share of children in total population. Then, with some lag, first in Europe around 1890-1920 period and later outside Europe around the 1950s, sharp declines in fertility were observed. These declines resulted in smaller sized age cohorts in successive generations, creating a “bulk” that corresponds to the baby boom generation. This whole process of population dynamics is called the demographic transition, which can be summarized as a transition from initially high levels of fertility and mortality rates and more or less stable populations to increasing population growth rates due to increasing life expectancies and still high fertility rates, and finally towards dramatically falling fertility rates accompanied by still lower mortality rates, which lead to graying populations with decreasing and even negative growth rates. Population aging, as a by-product of demographic transition, is a current phenomenon in most of the developed countries, and although developing countries, that are at earlier stages of demographic transition, have relatively younger populations they will also experience population aging in the coming future.

The demographic trends and projections from the 1950s onward give a good idea about how the main demographic indicators evolved (and are expected to evolve) in different regions of the world. To have a better understanding of the demographic transition processes experienced by different regions in the world and to identify the challenges and opportunities posed by these processes, it is imperative to look at these main historical demographic trends and projections. These trends will also provide guidance for some of the demographic experiments conducted in the rest of the thesis, as well as for the population debate mentioned in the rest of the chapter.

According to United Nations (UN) population statistics, the five year annual average growth rate of the world population is steadily declining since 1965-1970. Figure 2.1 reports the population growth rates for the world as a whole, along with the rates for more developed, less developed and less developed excluding least developed regions. [5] Although it is a more recent phenomenon for the less developed regions relative to more developed regions, the gradual decline in population growth rates over time is commonly observed in all regions. Population is projected to begin shrinking in more developed regions starting around 2030-2035. Despite an expected fall from 2.1% in 1950-1955 to 0.22% in 2045-2050, the growth rates in other regions of the world will remain above that in more developed regions throughout this period.

The economic implications of this ongoing transition in population growth rates are hard to predict without considering the evolutions of fertility and mortality rates separately. Considering these two demographic indicators together is important, because assuming away migration, the age structure of a population is determined by these two forces jointly, and two populations growing at the same rate may demonstrate different age structures. The life-cycle hypothesis first formulated by Modigliani and Brumberg (1954) implies that the age distribution of a population is a determining factor for aggregate saving and consumption, because different age cohorts behave differently due to different preferences and budget constraints.

[5] This regional classification is due to U.N.

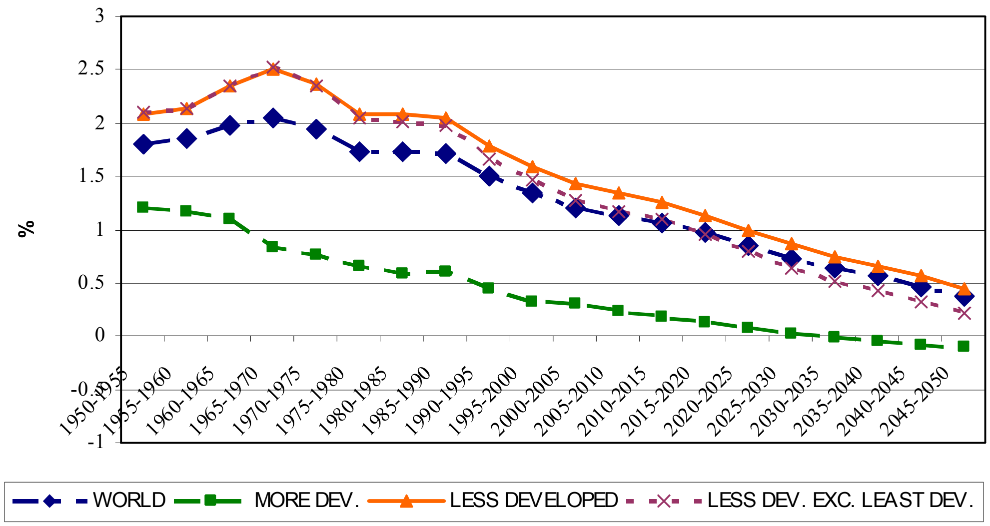


**Figure 2.1: Average Annual Population Growth Rates**

Therefore a thorough analysis of economic implications of demographic change requires the investigation of the impact of both life time uncertainty and fertility on economic outcomes. Figure 2.2 shows the total fertility rates[6] in different regions of the world. Especially until the 1970s, a big gap is observed between fertility rates of more and less developed regions, clearly indicating that demographic transition of more developed regions started earlier than the demographic transition of less developed regions. Despite the marked difference between total fertility rates in these regions in early decades, these rates are projected to converge to the replacement level of two births per woman in all regions by 2050.

The past and projected evolution of life expectancy is shown in Figure 2.3. As can be observed from the figure, there is a constant increase in life expectancy at birth in both more developed and less developed (excluding least developed) regions. In the

[6] Total fertility rate shows the average number of children a hypothetical cohort of women would have at the end of their reproductive period if they were subject during their whole lives to the fertility rates of a given period and if they were not subject to mortality. It is expressed as children per woman.

1950-1955 period, average life expectancies in the former and the latter were 66 and 42 years respectively.

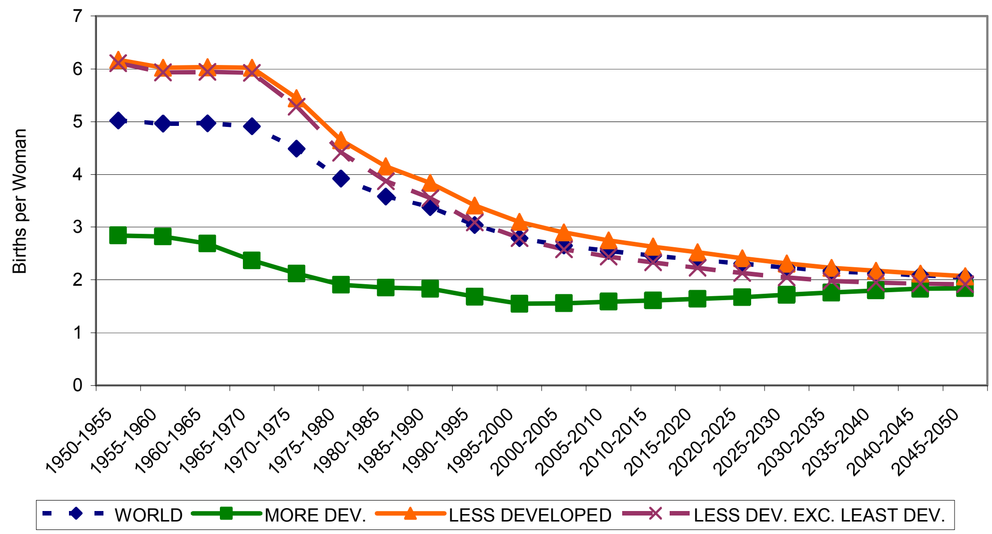


**Figure 2.2: Total Fertility Rates**

According to the medium variant of UN projections, the gap between life expectancies in more and less developed regions will continue to narrow down over the next 40 years but it will not disappear. The gap is expected to be around 6 years in 2045-2050 period. This observation hints a long-lived potential for transmission of the effects of demographic change between trading countries due to continuing mortality differentials.

The opposite is true for the share of young dependents. In more developed regions this share has been in an interval of 20%-30% until the 2000s and it is expected to remain below 20% at least until 2050. In less developed (excluding the least developed) regions, on the other hand, the share of young dependents has been around 40% in 1950s and is expected to fall down to 20% by 2050.

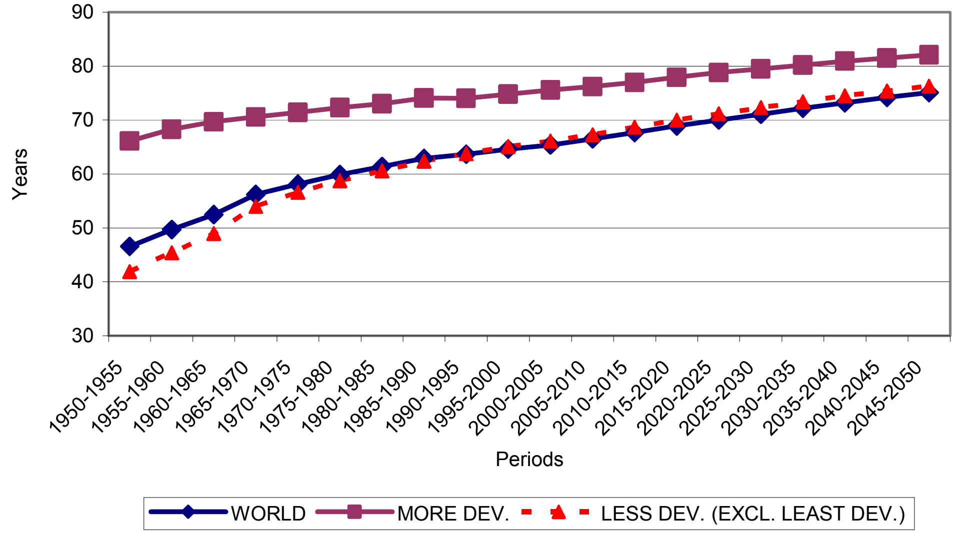


**Figure 2.3: Life Expectancy at Birth**

The resulting dependency ratios[7] are given in Figure 2.6. In the coming decades, child dependency ratio in more developed regions will remain more or less constant around 25%, while less developed regions will experience a significant decline in child dependency ratio due to falling birth rates and increasing share of working population. Around 2050, both ratios will be very close to each other near 30%. In contrast, the old dependency ratio in more developed regions will remain, as it has been in the past, above that in less developed regions. Around 2050, the former is projected reach 45%, whereas the latter is expected to be around 25%.

Looking at these demographic trends one can reach two main conclusions: The first conclusion is that throughout the next 50 years, child dependency ratios in more developed and less developed regions are likely to converge to a large extent, whereas both life expectancy at birth and old age dependency ratio in more developed

[7] Here child and old dependency ratios are defined as the ratios of population aged 0-14 and population aged 64+ to population aged 15-64 respectively.

countries will continue to be significantly above that in less developed regions; and this persistent difference in life expectancies calls for explicit modeling of adult mortality in open economy models that investigate the role of differential population dynamics as in this thesis.

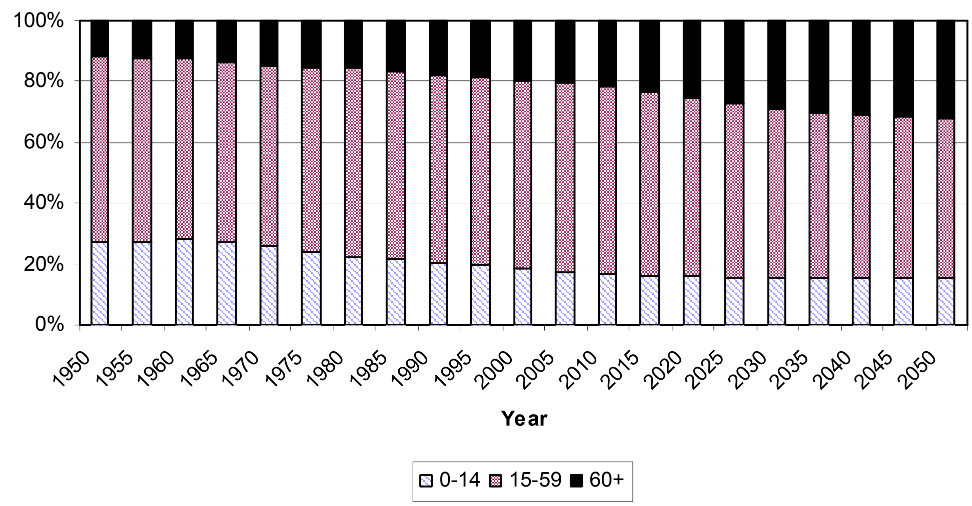


**Figure 2.4: Age Distribution in More Developed Regions**

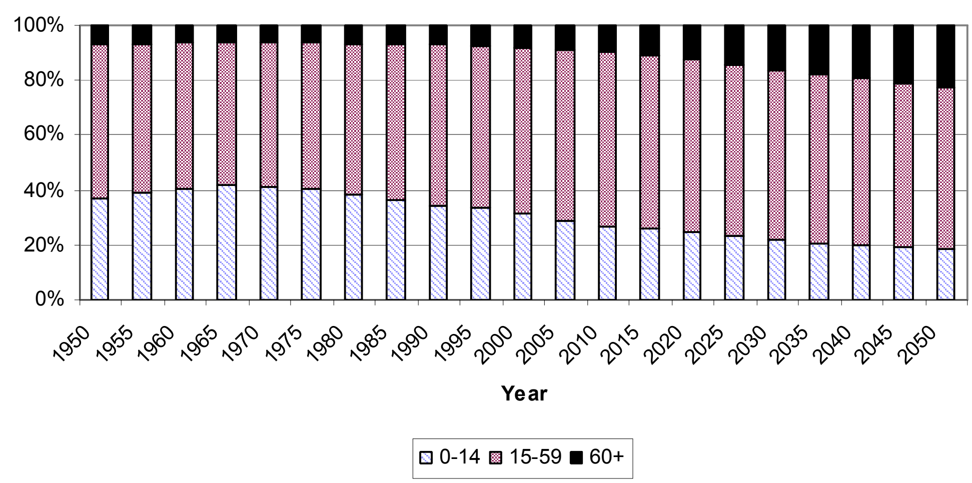


**Figure 2.5: Age Distribution in Less Developed (excluding Least Developed) Regions**

The second conclusion is that the decline in fertility rates in less developed regions, which began later than the decline in more developed regions, will continue to affect the economies in these regions.

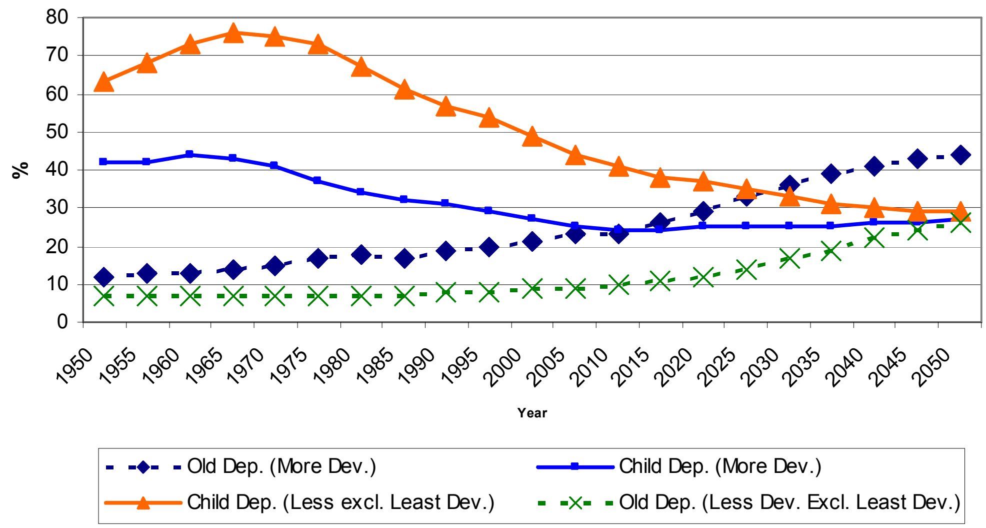


**Figure 2.6: Child and Old Dependency Ratios**

Today, average fertility rates[8] in more developed regions are below the replacement rate of two births per woman and they are expected to slightly and gradually increase back to the replacement rate. These conclusions suggest that not only demographic change within a particular country, but also demographic discrepancies among different countries are likely to play a critical role in determining the potential growth prospects of these countries.

Since Thomas Malthus' pessimistic work in 1798 *An Essay on the Principle of Population*, which mainly discussed that limited expandability of food supply relative to the growth in population size and the negative externalities due to rapid population

[8] Here, fertility rate refers to the total fertility rate.

growth may lead to an economic stagnation, also referred to as Malthusian trap, scientists and policy makers have been concerned with the economic consequences of demographic changes. During the period 1890-1930, a.k.a. the Malthusian Age, there have been studies that tried to address the issues first introduced by Malthus. As Spengler (1966) put it, 1930-65 was a period during which the previous misleading population forecasts were revised and various arguments were proposed to explain why Malthusian traps became irrelevant for the developed world. These arguments were mostly based on the observation that production technologies were increasingly favoring inputs other than land and natural resources thanks to rising investment in education, scientific discovery and applied technology. Despite technological advancements, looking at trends of his time, Spengler (1966) draws attention to the threat of food shortages and depleted natural resources that humankind may face in the remote future, unless fertility levels are not taken under control. He defends the need to reduce fertility rates through direct or indirect measures such as taxation on child related consumption, so that parents internalize all kinds of costs associated with reproducing and rearing children. Kelley (2001), in his survey on population debate from the beginnings of the 1950s to the first years of the 1990s, makes a distinction between ‘traditionalism’ and ‘revisionism’ as two approaches regarding the economic analysis of demographic change. The traditionalist approach was established and sustained over 1960s and 1970s by the Coale-Hoover framework introduced in Coale-Hoover (1958). This framework (1) concentrates on the direct and short- to intermediate-run effects, (2) ignores the feedback responses of economic agents to the first order effects of demographic change and (3) ignores the direct positive impact of population on output growth through scale economies. Simulating a mathematical model calibrated with Indian data, the authors conclude that lower population growth would significantly enhance per capita growth of the Indian economy. The book identifies adverse effects of rapid population growth on three different fronts: The first result is a reduction in capital-labor ratio (capital shallowing), the second result is a fall in saving rates caused by a rise in youth-dependency ratio and a corresponding increase in household consumption, and the third result is a diversion of investment from productive, growth-enhancing uses towards health and education expenses. Although Coale-Hoover (1958)'s formulation of the relationship between population and economic growth was consistent with the dominant growth paradigms of the time – which underlined the role of physical capital accumulation, after human capital formation, technological change and institutions started to be pronounced as primary sources of growth, Coale-Hoover framework's influence on population debate began to diminish towards 1980s. The revisionist approach, strongly voiced by Simon (1981), was a challenge to traditionalism. Unlike traditional methodology, it (1) focuses on the long-run effects, (2) accounts for feedback effects such as price-induced substitutions and (3) makes a balanced assessment of the connections between population and economic development, taking positive as well as negative effects of population growth into consideration. While almost all of the revisionist studies have found that slower population growth would improve the economic growth prospects of developing countries, unlike traditionalist studies, they have been less pessimistic about the extent of the negative net impact of rapid population growth on economic development due to their different methodology.

The main empirical results established during the early 1990s regarding the relationship between population and economic development in developing countries can be summarized as follows:

**1)** Population growth had no significant impact on per capita output growth during the 1960s and 1970s, whereas there was a negative impact on per capita output growth during the 1980s for a large number of both developed and less developed countries (see Kelley and Schmitt (1994, 1995)).

**2)** The level of development in the 1980s, as measured by per capita income level, has been determining for the direction of the effects of fertility declines. It is in the negative direction for less developed countries (LDCs) and positive for many developed countries (DCs).

**3)** Mortality declines positively affect growth prospects mostly by increasing the incentives to invest in human capital, although they are only partly responsible for high population growth.

**4)** Increases in population density and size, and increases in the relative size of the working age population are positively associated with economic growth, whereas increases in youth dependency ratio (share of the age group 0-15) are negatively associated with growth. This fact implies that, depending on the timing of demographic effects and the resulting age distribution, net impact on economic growth can be in either direction or neutral. (see Kelley and Schmitt (1994)).

**5)** Increasing elderly dependency ratio plays a very limited role in slowing down economic growth, whereas young dependent population, that does not work or save at all, significantly contributes to economic slow-down (see Bloom and Williamson (1998)).

**6)** Microeconomic studies reveal that increasing family size clearly reduces economic growth (see Cassen (1994)).

The literature on the economic implications of demographic change owed its rapid development after the 1980s primarily to two factors. One of them is the introduction of the OLG models into the literature of demographic economics, where different generations of agents coexist and engage in economic activity. The second one is the formulation of the life-cycle hypothesis, which distinguishes between the saving and consumption behaviors of agents at different stages of their lifetimes.

Aside from the growth rate of a population, its age structure highly matters in economic terms. As Bloom, Canning and Sevilla (2001) argue, the importance of age structure of a population has been mostly overlooked by scholars, who have usually concentrated on population growth solely as a determinant of economic growth.[9] The authors point to the fact that relative sizes of different age groups in a population matter, because different age groups behave differently with different objectives and capabilities. Based on this reasoning, scholars recognized that accounting for the heterogeneity of agents across different age cohorts is very crucial in understanding the effects of demographic change. OLG models that are originally attributed to Samuelson (1957) and Diamond (1965) were considered as very convenient tools for this purpose, and economists working in this field started to use them widely both in policy oriented and theoretical analyses. Tobin (1967) can be named as one of the first simulation studies that allowed for the coexistence of workers and retirees in the economy. It claims that increasing pace of population growth will increase the ratio of workers to retirees and thus raise the private savings rate in the economy.[10] However, Auerbach and Kotlikoff (1987) provided the first example of the contemporary OLG models designed for the economic analysis of demographics. In their seminal book,

[9] There are also economists who defend the "neutralist theory" that, let alone the age structure, even population growth has no significant effect on economic performance. For further information, see Bloom (2001).

[10] Note that if children were introduced into the model as dependents to workers, private saving rates would respond differently.

*Dynamic fiscal policy*, the authors present a model with changing fertility to compare the likely effects of demographic transition under different social security policies. The life-cycle hypothesis (LCH) proposed by Modigliani-Brumberg (1954, 1979) provides the rationale for using OLG models in the study of the economic impacts of demographic change. LCH claims that optimizing agents tend to smooth consumption over their life span. At childhood and young ages agents consume out of their parents income; in the early years of working-age, due to their limited earnings, they save less; in the middle phases and towards the end of their working-age their savings peak to cover their retirement-age expenditures. When retired they either dissave or considerably reduce their savings to meet their consumption. Combined with the age structure of a population at a certain point in time, this life-cycle effect on production, savings and consumption of individuals results in aggregate macroeconomic measures that depend on the composition of population. Therefore, LCH lies at the core of the argument that co-existence and interaction of different generations in the economy imply different consumption and saving behaviors throughout time, as the age structure –more specifically relative sizes of working, retired and youth populations– evolves with time. However, as it is often the case, there have been skeptical views about the validity of LCH such as those expressed by Deaton (1991, 1992), Carroll-Summers (1991), Carroll (1992, 1997) and Carroll-Samwick (1997). These authors mainly argue that empirical evidence from micro-data shows that LCH does not confirm the declining saving ratios in the US during the last decades and the significant rise of saving ratios in some Asian countries. Moreover, cohort specific savings data computed using household surveys are also at odds with LCH, since they find that average saving rates of 70-74 and 65-69 cohorts are 15.1 % and 15.4 % respectively. These rates are slightly above the average saving rate of 40-44 cohort. However, it should be noted that some particular regulations regarding the pay-as-you-go pension programs greatly distort the underlying age-earning profile in the economy by changing the age for early retirement eligibility. Miles (1999) discusses that the mismatch between the age-saving relation suggested by LCH and the one estimated by using only household data may largely be due to the mis-measurement of pension income.[11] Indeed, there are studies such as Bosworth et al. (1991) and Alessie et al. (1995) which found that under pension adjustments saving rates of households moving through retirement ages turn out to be significantly lower than under no-adjustment case. Despite the challenges against it, LCH is extensively used in demo-economic studies both in its simple and sophisticated formulations. Despite the sensitive nature of their results to parameter selection, as Miles (1999) argues, using calibrated OLG models is advantageous over doing cross section analysis using micro data or time series analysis for analyzing the impact of demographic shifts. Attaining reliable results by using micro data is difficult due to the lack of information on pension wealth at household level and absence of micro data that cover a sufficiently long time period. Time series analysis of demographic shifts does not promise reliable results either, due to the absence of time series data that is long enough to capture necessary variability in the age structure of population. Especially in policy oriented forecasts of the effects of future demographic changes, OLG models have clear advantages over time series analysis of the relation between savings, capital accumulation and demographic structure. First, they are more theory-based and can be used in policy simulations; second, they can be adjusted for varying degrees of foresight, rationality and other behavioral patterns of individuals by changing parameters governing the rates of time preference, risk aversion and elasticity of substitution among goods.

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[11] See Miles (1999) for further discussion.

We can identify two main currents in the literature of economic demography that make use of OLG framework, but differ in their motivations in using this framework. One current focuses on the study of population dynamics and economics from a historical perspective. This perspective tries to provide economic explanations for the past demographic stages in the history of humankind and the transition from one stage to another. This necessitates OLG models that have fertility as a choice variable. The second current is concerned with the study of the impact of population dynamics on macroeconomic measures. This perspective utilizes simulation models calibrated for particular countries or regions and that treat fertility as an exogenous variable. It is primarily interested in predicting the future effects of economic policies, given the current projections for demographic trends. Unlike the former current, this line of research is one-sided, given the lack of causality from economics to demography. It looks at several aspects of the demographic impact on economies. Pressures on social security systems, fiscal sustainability, infrastructural investments, health and education systems are some of these aspects.

OLG models are widely used in evaluating different policy or population scenarios. These simulation studies are often country or region specific analyses. Some focus on various issues related to the problem of aging mostly faced by developed industrial countries and others look at issues related to the opportunities and problems posed by the increasing share of working age population in developing and less developed countries. Auerbach and Kotlikoff (1987) is the first example of extensive OLG treatments that incorporate population dynamics. The model presented in the book analyzes the impacts of two different demographic transition scenarios -a sudden and permanent reduction in the birth rate (baby bust) and a cycle of decline and rise in the birth rate followed by a permanent fall (bust-boom-bust)- under the absence and presence of social security and under different fiscal regimes. This study was followed by an OECD Working Paper by Auerbach et al. published in 1989. The paper addresses important policy questions regarding social security policies and assesses the impact of changing dependency ratios on saving rates and wage rates for the case of four OECD countries: Germany, Sweden, United States and Japan. The model used improves upon the one used in Auerbach and Kotlikoff (1987) by incorporating technological progress, bequest behavior, possibility of international trade and government consumption expenditures that depend on the changing age composition. The major contribution of this study is its manifestation that general equilibrium adjustments may offset part of the adverse effects of increasing dependency. The results for base scenario, where average replacement rates, initial ages for receiving pension benefits and pattern of public spending are assumed to be fixed, roughly suggest a fall and a subsequent rise in national saving rates which in the long run converge to lower rates than the initial levels for all countries. Real wage rates mostly follow a rising trend during the transition and converge to levels higher than their initial steady state levels, except for Sweden. Later on, Miles (1999) analyzed the case of UK and the rest of Europe by using an OLG model similar to Auerbach and Kotlikoff (1987). However, unlike in the latter, the author allows for technical progress by arguing that technological changes affect earning profiles of future generations, which in turn influence the asset accumulation pattern over the life cycle. Moreover, the simulations employ an age-productivity relation that is based on micro evidence from UK. The simulations are run for the time period 1960-2060, and the feedback effects from policy to longevity and fertility are assumed away. The results for the base scenario involving a PAYG pension regime with a constant replacement rate of 0.2 suggest that falling fertility rates cause swings in saving rates and real interest rates that eventually end up at lower levels than initial levels confirming the results of Auerbach et al. (1989). Fougère and Mérette (1998) extend the macroeconomic analysis of population aging to seven OECD countries (Canada, France, UK, Sweden, Japan, USA and Italy). A demographic shock in the form of falling birth rates is simulated starting from 1954 under the assumption that birth rates converge to the replacement level at steady state. Like in previous studies, saving rates in the seven countries decline at differential rates and partly recover around 2050s towards a lower level than the starting level. Wage-income tax rates on the other hand gradually rise reaching their peak point around 2050s and converge to higher rates than initial rates. In 1999, Fougère and Mérette introduced the feature of endogenous growth through human capital accumulation to their 1998 model. In this framework, population aging affects economic growth not only through changes in savings and labor force, but also through changes in human capital and R&D investments. In line with the life cycle hypothesis, young generations invest heavily in human capital, whereas middle aged generations invest heavily in physical capital. Like in previously mentioned OLG studies, population growth is exogenous, unlike in endogenous fertility models. At the bottom line, this study shows that introducing human capital formation into the model significantly changes the economic outcomes of demographic change by making a tradeoff between physical capital and human capital in response to population aging possible. The results differ from the previous study without endogenous growth feature in different respects: As population aging kicks in, human capital investments begin to rise resulting in a bigger reduction in effective labor supply, saving rates and per capita output in the short-run, but as the transition proceeds, an increase in effective labor supply offsets initial decline and results in higher real per capita output in the long run contrary to the finding of the original model. Despite significantly declining saving rates, increasing long run economic growth is explained by a shift of investment from physical capital to human capital. A caveat of the model that can be quantitatively important is the lack of age dependency related government expenditures such as education and health care, which are present in Auerbach et al. (1989). As an extension to Fougère and Mérette (1999), Sadahiro and Shimasawa (2002) endogenize the human capital accumulation mechanism by introducing time invested to schooling as a decision variable in the utility function, which together with the level of per labor physical capital determines the level of human capital for an individual. Two demographic scenarios -positive and negative population growth- are simulated which provide the following qualitative results: (1) While the population size is shrinking, individuals have more tendency to allocate time to schooling to benefit from increasing returns to human capital in the face of increasing capital-labor ratio. (2) Labor supply and human capital are negatively correlated.

As population aging process intensifies and the old-age dependency ratios increase, developed countries will be increasingly facing fiscal problems related to the provision of social security and health services, and dampened economic growth related to possibly lower saving rates and investment. Under these circumstances, questions about the real extent of the economic burden that will be caused by aging and about how to cope with this burden become important. In the beginning of the 1990s, extensive research projects were initiated to predict the welfare effects of population aging and find solutions to cushion the adverse effects of aging.[12] Scholars came up with several policy measures involving pension and health system reforms, measures for sustainable public finances and structural reforms aiming at higher technological progress that would improve total factor productivities. It has been a rather recent phenomenon that scholars start to investigate the welfare implications of cross-border economic relations among countries with different population dynamics. This new strand of literature focuses on open economy implications of demographic change through various channels like trading of goods, international capital markets and labor movements (migration). The main question that motivates this research agenda is whether close economic ties among countries with different age compositions will help to digest the heavy burden of population aging in the western world. Put differently, will economic globalization mitigate or deepen the current and future problems in graying nations, if it will have any significant effect at all? Looking from the opposite angle, some scholars try to answer the question of how, if ever, developing countries with lower dependency ratios can benefit from their demographic windows of opportunity before it is too late. Do demographic spillovers through international capital flows and trade with developed countries benefit or hurt the developing and less developed countries?

Before addressing these issues, however, the question of whether current economic interdependencies and linkages among nations are sufficiently strong to leave room for the effects of population dynamics to be transmitted from one country to the other, should be given a convincing answer. The famous Feldstein-Horioka (1980) puzzle

[12] As relevant studies in this area, see David Wise (1992, 1994, 1998); the World Bank volume on *Averting the Old Age Crisis*; Richard Disney (1996); Axel Borsch-Supan (1996); IMF Occasional Paper by Sheetal Chand and Albert Jaeger (1997); Richard Kohl and Paul O'Brien (1998).

suggests that there is a strong correlation between national savings and domestic investments. Some scholars interpreted this result as an evidence of imperfections in international capital markets forcing domestic investments to be almost entirely financed by domestic savings. Later on, however, some theoretical models showed that even under the presence of a barrier-free international capital market, Feldstein-Horioka type of a correlation between domestic savings and investments might emerge.[13] This, however, does not imply that imperfections do not exist. Both international capital and commodity markets have imperfections, though the former less so. Despite these imperfections, the main point here is that Feldstein-Horioka puzzle is no longer considered as convincing evidence against the presence of cross-border effects of demographic shifts.

In the literature on open economy effects of demographic change, some of the studies are concerned with predicting the future effects of demographic changes upon different economic variables such as per capita GNP, saving rates, interest rates, exchange rates and consumption. For this purpose, model builders exogenously feed in paths for fertility and mortality rates based on demographic projections into their models. Not only this, they also feed in large sets of technology and cohort-specific preference parameters into their models to attain as realistic results as possible. These policy oriented comprehensive studies either take a multi-country OLG approach in modeling the economy or "shortcut" approaches that allow the aggregation of the behavior of different age-cohorts. It is still an unresolved debate in the literature, which type of model provides more promising results. On the one hand, there are the proponents of multi-cohort general equilibrium OLG models, who argue that agent

[13] See Obstfeld and Rogoff (1996, 2001) for various interpretations of Feldstein-Horioka findings.

heterogeneity is a very crucial element for properly capturing the effects of demographics on the economy. On the other hand, there are studies such as, Masson-Tryon (1990), Fair-Dominguez (1991), Meredith (1995), Faruqee (2003) which devised different "empirical shortcuts" to derive an aggregate consumption function so as to avoid dealing with consumption decisions of each cohort as in multi-cohort OLG models. Multi-cohort models explicitly keep track of different cohorts with their different saving and consumptions decisions. However, as Bryant and McKibbin state in a Brookings Discussion paper, although these models account for different preferences of different age groups in the economy, they are highly complex and involve a very demanding theoretical treatment especially when there is a multitude of countries and currencies. The approach of modeling through "empirical shortcuts" relies on models that are similar to IMF's MULTIMOD[14]. MULTIMOD is a dynamic macroeconometric model to simulate the effects of policy shifts in a multi-country open economy setting. It is a general equilibrium model that generates both short- and long-run effects of a shock to the system. There have been several studies that made use of MULTIMOD or similar models to simulate the transitional impacts of demographic shifts. One salient property of these models is their adoption of a framework that contains two complementary assumptions. First one was introduced by Blanchard (1985), where agents are assumed to face a constant-throughout-life probability of death $p$ so that under continuous time, the expected length of life becomes $\frac{1}{p}$, which can be easily adjusted according to the demographic data on life expectancies. Second assumption draws on Yaari (1965) by allowing agents to make

[14] MULTIMOD is defined as "a modern dynamic multi-country macro model of the world economy that has been designed to study the transmission of shocks across countries as well as the short-run and medium-run consequences of alternative monetary and fiscal policies".

costless annuity contracts with insurance companies contingent on their deaths. As the authors argue, this is an important technical “shortcut” that enables the model builder to derive an aggregate consumption function that can implicitly keep track of consumption and wealth of individual cohorts. A critical assumption in this type of models was that individuals face a constant probability of death throughout their lives. Faruqee (2003) succeeds to relax this assumption by incorporating age-specific mortality rates into an overlapping agents model of aggregative type through a hyperbolic function that closely replicates Gomperty’s Law on age specific death rates. One should note that these aggregative policy simulation models all assume a negative relationship between saving rates and total dependency ratios.[15] At this point it should be noted that the particular characterization of the relationship between population dynamics and saving in a model plays a very crucial role for the results attained by that model. In general, the findings indicate to the presence of an effect of population on savings, but there is a controversy on the magnitude of these effects. For instance, Williamson and Higgins (1997) look at the case of Taiwan and find a very large swing in savings by using an OLG model whose life-cycle features are derived from the analysis of pooled aggregate saving data. Lee et al. (2001) use a simulation model for saving based on a life-cycle framework. They also aim to assess the impact of demographic transition on wealth accumulation and aggregate savings in Taiwan and some other countries. The study underlines the importance of mortality declines in the historical upward movement of saving rates due to an increasing retirement period. Their simulations roughly suggest a swing in saving rates, i.e., the saving rates constantly increase, reaching a peak point around early years of the 2000s, which is

[15] The total dependency ratio is the ratio of the sum of the population aged 0-14 and that aged 65+ to the population aged 15-64.

followed by a fall in the rates as the overall dependency ratio increases. This swing is smaller in magnitude and comes somewhat later relative to the swing suggested by Williamson and Higgins (1997). Moreover, the authors show that an individual support system instead of a wealth transfer system (such as a pay-as-you-go pension scheme) and a modern demographic regime (higher life expectancy and lower fertility) instead of a traditional regime can raise the saving rates to a relatively higher level. Finally, they find that the speed of demographic transition matters for the profile of saving rates, where rapid transition leads to a higher and earlier swing and gradual transition leads to a moderate swing in saving rates. Deaton and Paxson (1997, 2000) is a micro-based study where data from annual National Family Income and Expenditure survey is used to construct age profiles of income, consumption and saving. Analyzing the case of Taiwan, while the 1997 paper finds no evidence for the effects of demographic change on saving, the 2000 paper suggests a very limited role for demographic change. In summary, although it is commonly accepted that age structure of the population and in particular the dependency ratios have a bearing on saving rates, the magnitude and timing of the effects is open to debate.

The results of open economy simulation studies on demographic change can be classified into two based on their selection of regions or countries of analysis. While some studies focus on the implications of trade and capital mobility among developed countries, which are more or less at the same stage of demographic transition, some others analyze the demographic transmissions between developed and developing countries that are at different stages of the transition. The earliest one among the OLG studies that incorporate an open economy framework into its analysis is Auerbach et al. (1989), which introduced the possibility of international trade in the analysis of demographic shifts under the assumptions that all of the four OECD countries are small relative to the rest of the world and labor is immobile across countries. Findings suggest that saving rates –particularly that of U.S. and Japan- remain above their closed economy levels throughout the transition, which is in line with the fact that capital outflows towards the rest of the world[16] prevent the real interest rates from falling and thereby create a higher incentive to save. Like the previous one, Masson and Tryon (1990) also tried to gauge the macroeconomic consequences of population aging in industrial countries. As different from Auerbach et al. (1989), instead of using an OLG framework, this study uses an aggregate consumption function à la Blachard (1985) which depends on two sources of wealth, i.e., human capital ($H$) and government bonds ($B$), and a marginal propensity to consume out of total wealth which depends on the probability of death and the rate of time preference. Their methodology involves the OLS estimation of the relationship between the percentage changes in overall dependency ratio and aggregate consumption and using MULTIMOD to simulate the combined effects of increasing dependency ratio through the channels of reduced saving rates, smaller labor force and higher government spending. The study suffers from the important shortcoming that the estimated response[17] of consumers' behavior to changes in dependency ratio is assumed to be a good guide for the distant future[18], although the share of old population will be substantially higher in that future. Some scholars have looked for an explicit answer to whether international linkages may offset the negative effects of aging. Such a study is due to Fehr, Jokisch and Kotlikoff (2003, 2004) which questions the role of capital flows and immigration as two

[16] Due to small open economy assumption, the rest of the world has the potential to absorb all the capital outflows, precluding any feedback effects.

[17] The data used in the estimation covers 1969-1987 period across several industrial countries.

[18] 2025 is the terminal year of the simulation experiment.

potential cushions against the adverse effects of demographic transition by simulating a three region life-cycle model –involving US, EU and Japan– with traded goods and capital. It allows for immigration, age-specific fertility, mortality decline, life span uncertainty and intra-cohort heterogeneity. The authors found that aging will have great damage on these regions by crowding out their capital stocks, lowering real wages and raising real interest rates. The capital in the region with a less intense aging process will flow to the regions with more intense aging process. More importantly, immigration does not seem to be a sufficient remedy for aging related problems in all regions unless there is a mass immigration of high skilled workers from underdeveloped world. The paper argues that among all other policy alternatives in dealing with population aging the most plausible one is the elimination of current pension systems and financing the accrued pension liabilities with consumption tax.

Attanasio-Violante (2000) is one of the earliest studies that take the OLG approach to analyze the effects of demographic transition on two regions that go through different stages of the demographic transition (the economy of the region with faster transition is calibrated to resemble the US and European economies –North–, whereas the other region's economy is calibrated to resemble Latin America) under a general equilibrium framework. This open economy study found that over the past 50 years, the demographic transition could have caused an increase of about 0.5 % in per capita income per year in Latin America and 0.3 % in North, controlling for other factors. Again two relatively early studies, that incorporate trade into an OLG GE model with different population growth rates in trading countries, are Sayan-Uyar (2001) and Kenc-Sayan (2001). They establish the possibility that unequal population growth rates and/or dependency ratios between two otherwise identical trading countries are sufficient to cause trade flows by altering relative labor and capital endowments and inducing wage and rental rate differentials. The first study also motivates migration from high population growth country to slow population growth country as a result of wage differentials. The second study addresses the issue of demographic transmission from EU onto small countries, where increasing elderly-dependency ratio in the former will alter the relative prices of traded and non-traded goods and the sectoral allocation of resources in the latter by imposing its own terms of trade upon the latter. Another demographic transmission channel would be due to changing factor proportions, more specifically capital-labor ratios in the large industrial countries that undergo a fast aging process. It is a common prediction that capital-labor ratios in developed countries will rise further as population aging intensifies. Then the western world will increasingly become a net exporter of capital and capital intensive goods. These potential capital inflows from developed countries to developing countries constitute an opportunity for the latter to stimulate economic growth, given that necessary environment to attract and retain foreign investment is provided.

Some scholars in the literature called for the need to separately account for the effects of youth- and elderly dependency ratios on the economy. Departing from the observation that the youth dependency ratio should be as important as the elderly dependency ratio in determining the macroeconomic effects of demographic shifts, Bryant et al. (2004) investigate the presence and extent of spillover effects of demographic change through exchange rates, external sector imbalances and international capital flows. They introduce reasonable components to their two country open economy model such as variant and invariant parts of child support to their model under which changes in youth dependency ratio magnify the macroeconomic responses. They simulated the effects of two demographic scenarios that hit the two countries (US and a hypothetical country ZZ) asymmetrically: a sharply falling birth rate, which partly recovers through a cyclical behavior before reaching its steady state level, hits ZZ, whereas the US economy faces a gradually declining birth rate without a cyclical behavior that converge to the same steady state level as of ZZ. The observed effects on the real interest rates of the two economies suggest that ZZ experiences sharper declines in real interest rates compared to US, but both responses are quite cyclical. However, falls in real interest rates are less pronounced in ZZ and more pronounced in US relative to their responses observed in the respective closed economy cases. This implies that the effects of asymmetric demographic shocks on the country with the stronger demographic shift are damped and on the country with weaker demographic shift is magnified with respect to their relative autarky effects.

One widely overlooked issue in the literature is the welfare implications of differential population dynamics for trade in a dynamic framework. Sayan (2005) is a valuable contribution at this front, since it provides a well-founded explanation for why in a dynamic setting, international trade[19] may not lead to Pareto superior outcomes relative to autarky: Using a two-country, two-factor, two-commodity and two-generation OLG model, the paper shows that the country with lower population growth rate (which can be thought as representing today's developed countries) may become worse off compared to the autarky. There had been previous studies that questioned the validity of what static trade models suggest about the welfare implications of trade under a dynamic setting. Stiglitz (1970) and Deardorff (1973) are early examples of how trade may be detrimental for the long run welfare of society when time dimension

[19] The paper uses the Hekscher-Ohlin framework in a dynamic setting.

is involved. They showed that in a convex international trade model, trade may actually reduce the steady state level of per capita consumption in the economy. Similarly, Kareken and Wallace (1975, appendix) showed that all future generations in a country may become worse off due to the opening of the economy to trade in consumption goods. Fried (1980) elaborated on this argument by using a simple two-generation (young and old) two-factor (labor and land in fixed supplies) OLG model. He showed that if intergenerational transfers are inapplicable or too costly to carry out, then moving from autarky to free trade in consumption goods may reduce the welfare of all generations after the regime shift, while only making the old generation that was alive at the time of regime shift better off. This result is due to the fact that domestic relative output prices change to converge to the prices prevailing in the rest of the world, which in turn affect the relative factor prices. If this effect leads to a fall in real wages and if the negative income effect of this fall cannot be totally offset by the increasing value of the consumption bundle, then future generations indeed lose from this regime shift when their welfare is compared to the one in autarky. Fried (1980)'s demonstration was based on a simple and specific model, where supplies of both labor and land were fixed[20], i.e., there is no accumulation in the factors of production. Moreover, his analysis assumed that relative prices of outputs were directly imposed by the rest of the world. To gain more insight about the implications of dynamic international trade, a more general OLG framework was needed that would allow accumulating factors in production technology, involve multiple sectors producing goods with different factor intensities and be applicable to trade models such as HO framework, where world relative prices are determined jointly by trading parties. Galor (1992) opened the way

[20] Young and old populations are assumed to be equal and constant throughout time and the fixed amount of land is assumed to be imperishable.

for such analysis by devising a two-generation, two-sector and two-factor OLG model and deriving the sufficient conditions for the global uniqueness of a perfect foresight equilibrium[21] of this model. Besides, he showed the theoretical possibility of multiple non-trivial steady state equilibria given that the investment good is capital intensive. This paved the way to the derivation of important results in dynamic trade literature. One of these results is due to Mountford (1998), which contributed to the dynamic trade literature by demonstrating that in the long run, welfare results in a standard convex HO model may be contrary to those implied by the static version of the model. Using a two-generation, two-sector OLG model, Cremers (2005) investigates the dynamic effects of international commodity trade. She finds that among the two trading countries the one with higher saving rate experiences a slowdown in capital accumulation relative to autarky, whereas the one with lower saving rate is better off under free trade.

Another less discussed issue in the open economy literature is the influence of political process on economic decisions as a result of changing demographics. One of the rare studies on this issue is due to Tosun (2003), which looks at the impact of voting behavior of an aging population as the determinant of the government's taxation policy in a two countries open economy setting with capital mobility.[22] Through a very simple two generations OLG model, it demonstrates that two countries may diverge in terms of economic growth through diverging fiscal policies as a result of differing population growth rates.

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[21] Galor (1992) showed that unlike in the case of pure exchange economy, in a production economy the gross substitutability of consumption goods is not sufficient for global uniqueness of the perfect foresight equilibrium. In addition, the investment good should be relatively capital intensive and the second period good should be a normal good for global uniqueness.

[22] Beetsma (1995) is such a study whose political economy feature is very similar to the one in Tosun (2003), but it uses a closed economy model unlike Tosun (2003).

# CHAPTER 3:

# The Basic Model

The basic model is an infinite horizon, discrete time OLG model that involves three generations –children[23], working adults and retired elderly-, and two sectors, each producing a single good. Good 1, can be viewed as an aggregate good composed of everything a child needs and is consumed only by children, whereas good 2 is only consumed by adults and elderly, and also serves as the investment good in the economy as in Sayan (2005) and Jelassi (2004). Good 1 is assigned as a child specific consumption so that the model can give better insights about the impact of different types of demographic shocks on the consumption spending for children. This formulation is in line with the real life facts, because goods such as toys, child care products and services such as different levels of education are mostly consumed by this generation. The amount of good 1 consumed by each child, the amount of good 2 consumed by each adult and the amount of good 2 consumed by each old agent at time t are denoted by $c_{1t}$, $c_{2yt}$ and $c_{2ot}$ respectively, where the subscripts $y$ and $o$ stand for the members of young (adult) and old generations. Child care good (good 1)

[23] The word child here can be misleading, because the child generation as defined here rougly covers the agents with 0-25 years old.

is assumed to be relatively labor intensive and the consumption-investment good (good 2) is assumed to be relatively capital intensive. The market for goods, capital and labor are assumed to be perfectly competitive. Therefore consumers and producers take prices, wage rates and rental rates as given. Capital and labor are perfectly mobile across the two sectors. The agents are in all periods homogenous within each generation. Each adult living at time t bears $n_t$ babies at the beginning of his adulthood and faces a probability of survival $\lambda_t$ into old age, i.e., to period t+1, where $n_t$ and $\lambda_t$ are exogenously given. Agents are assumed to have perfect foresight except for the time of their death. In other words, the model assumes that probability of survival cannot be foreseen by agents with perfect accuracy. Thus, the intertemporal allocation of consumption is not affected by the mortality rate that the agents actually face. However, in this basic model it is assumed that probability of survival is not anticipated by agents. Thus, the inter-temporal allocation of consumption is not affected by the mortality rate that the agents actually face. This assumption will later on be replaced by perfect anticipation assumption, where agents will know the exact probability of death in advance and incorporate this information in their decisions. Since the model involves mortality, unlike Sayan (2005) and Jelassi (2004), agents who cannot survive into old age leave unintentional (or accidental) bequests to the next generation.

## 3.1 Population Dynamics

The population dynamics in this three generations OLG model are governed by birth and death rates, whith migration assumed away. Birth rate $n_t$ at time t shows the number of children born by each adult living at time t. $\lambda_t$ is the survival rate which shows the probability that an adult living at time t will survive into the next period where she will be old. Therefore $1-\lambda_t$ represents the death rate among adults at time t. The model does not involve child mortality simply because the impact of a change in child mortality can be easily captured by a corresponding change in the birth rate. The adult survival rate $\lambda_t$ does not affect the size of the labor force, but only affect the size of the old age population. $C_t, N_t$ and $O_t$ denote the child, adult and old populations respectively, and their evolution throughout time is given as follows:

$$C_t = N_t n_t$$

$$N_t = C_{t-1} = N_{t-1} n_{t-1}$$

$$O_t = N_{t-1}\lambda_{t-1}$$

Then, the growth rate of the total population is given as

$$\begin{aligned} g_t &= \frac{C_t + N_t + O_t}{C_{t-1} + N_{t-1} + O_{t-1}} - 1 = \frac{C_{t-1} n_t + N_{t-1} n_{t-1} + N_{t-1}\lambda_{t-1}}{C_{t-1} + N_{t-1} + N_{t-2}\lambda_{t-2}} - 1 \\ &= \frac{N_{t-2} n_{t-2}\left(n_{t-1} n_t + n_{t-1} + \lambda_{t-1}\right)}{N_{t-2}\left(n_{t-2} n_{t-1} + n_{t-2} + \lambda_{t-2}\right)} - 1 = \frac{n_{t-2}\left[n_{t-1} n_t - \left(1-\lambda_{t-1}\right)\right] - \lambda_{t-2}}{n_{t-2}\left(1+n_{t-1}\right) + \lambda_{t-2}} \end{aligned} \tag{3.1}$$

If both birth and death rates are constant over time, population growth is only determined by birth rates. In other words, the death rates do not affect the population growth rate as long as they remain constant. This fact can be seen by setting $n_t = n,\ \lambda_t = \lambda,\ \forall t$ and rewrite the population growth given above as

$$\frac{n^3 - n + \lambda\left(n-1\right)}{n\left(n+1\right)+\lambda} = \frac{\left(n-1\right)\left[\left(n+1\right)n + \lambda\right]}{\left[\left(n+1\right)n+\lambda\right]} = n-1 \tag{3.2}$$

Moreover, the old-age and child dependency ratios $E_t$ and $Y_t$ can be written as

$$E_t = \frac{O_t}{N_t} = \frac{\lambda_{t-1}}{n_{t-1}} \text{ and}$$

$$Y_t = \frac{C_t}{N_t} = n_t$$

The child dependency ratio is governed only by birth rates, while elderly dependency ratio at time t is governed by the ratio of survival rate and birth rate of the previous period.

When birth and death rates are not constant throughout time, then using (3.1) it is easy to show that

$\frac{\partial g_t}{\partial \lambda_{t-2}} < 0$ when $g_t > -1$, $\frac{\partial g_t}{\partial \lambda_{t-1}} > 0$, $\frac{\partial g_t}{\partial n_{t-2}} > 0$, $\frac{\partial g_t}{\partial n_{t-1}} > 0$ and $\frac{\partial g_t}{\partial n_t} > 0$.

Note that $\frac{\partial g_t}{\partial \lambda_{t-2}} < 0$ because $\lambda_{t-2}$ is positively related to the size of the old population at time t-1, which in turn is negatively related to population growth at time t as shown in (3.1).

## 3.2 Consumer Preferences

Each adult maximizes her lifetime utility $U(n_t c_{1t}, c_{2yt}, c_{2ot+1})$ by choosing the amount of good 1 to be consumed by her children, the amounts of good 2 to be consumed by herself in the current period and in the next period when she is old. Each adult inelastically supplies one effective unit of labor $l$ which is divided between the two sectors so that $l_{1t} + l_{2t} = l$ and earns a wage of $w_t$ per effective unit of labor supplied. Adults allocate their wage income between consumption and savings. $p_t$ is the relative price of good 1. The number of children born at each period is given exogenously. Children have no control over their consumption decision and are totally dependent upon their parents. The utility function is increasing and concave in both consumption goods. Old people finance their consumption out of their savings which they optimally chose in the previous period so that $c_{2ot+1} = (1+r_{t+1})s_t$ where $s_t$ is the per adult savings decided at the end of period $t$, and $r_{t+1}$ is the rental rate for capital that prevails at time $t+1$. Each adult solves the following problem at time t:

$$
\begin{aligned}
&\underset{c_{1t}, c_{2yt}, c_{2ot+1}}{Max} \; U(n_t c_{1t}, c_{2yt}, c_{2ot+1}) \; s.t. \\
&n_t p_t c_{1t} + c_{2yt} + \frac{c_{2ot+1}}{(1+r_{t+1})} = w_t l + (1-\lambda_{t-1}) \frac{N_{yt-1}}{N_{yt}} (1+r_t) s_{t-1} \qquad (3.3) \\
&c_{1t}, c_{2yt}, c_{2ot+1} \geq 0
\end{aligned}
$$

where the second term in the right hand side of the budget constraint is the amount of accidental bequests left by the adults of period t-1 who could not survive into period t.[24] Here unintentional bequests are a consequence of the uncertainty about the timing of death. Those adults who cannot survive into old age cannot consume the principal plus any accumulated return on their savings, and these returns are equally divided among the members of the next generation.[25] Per worker savings in the economy can be written as

$$
s_t = w_t l + \frac{(1-\lambda_{t-1})}{n_{t-1}} (1+r_t) s_{t-1} - n_t p_t c_{1t} - c_{2yt} \qquad (3.4)
$$

[24] One may argue that if working agents know that they face a non-zero probability of death, then they would incorporate this fact into their saving decision. After all, optimizing agents would prefer to consume more in the current period, if there is a possibility that they will not be able to consume out of their savings in the next period in the case of death. However, if the optimization problem is modified to address this problem, then adults deciding on their current and next period consumptions according to their expectations over the probability of death may run budget deficits in their last period of life, since the model does not allow old agents, who are sure to die at the end of the period, to borrow resources. To deal with this problem, I used the “chance-constrained programming” approach adopted in Yaari (1965), which required the probability of running a budget deficit to be zero. In other words, agents face a budget constraint where they need to allocate consumption as if they will be able to realize it with probability one.

[25] Although the assumption about the equal division of unintentional bequests is not very realistic, it is necessary for conducting a representative agent analysis. Hubbard and Judd (1987) and Pecchenino and Utendorf (1999) likewise assume uniformly distributed bequests in their treatment of unintentional bequests.

This equation says that what is not consumed by children and adults out of wage income and received bequests at the current period is saved for the next period.

We assume a Cobb-Douglas type inter-temporal utility function where adults receive utility both from their own consumption and from consumption of each child they bore. Thus, it is implicitly assumed that adults are altruistic towards their offspring, but when old, they only receive utility from their own consumption. Accordingly, the problem in (3.3) becomes:

$$\underset{c_{1t},c_{2yt},c_{2ot+1}}{Max}\left[\left(n_t c_{1t}\right)^{\theta}\left(c_{2yt}\right)^{1-\theta}\right]^{\mu}\left[c_{2ot+1}\right]^{1-\mu} \ \ s.t. \tag{3.5}$$

$$\begin{aligned} & n_t p_t c_{1t} + c_{2yt} + \frac{c_{2ot+1}}{(1+r_{t+1})} = w_t l + \frac{\left(1-\lambda_{t-1}\right)}{n_{t-1}}\left(1+r_t\right)s_{t-1} \\ & c_{1t} \geq 0,\ c_{2yt} \geq 0,\ c_{2ot+1} \geq 0 \end{aligned} \tag{3.6}$$

where $\theta$ is the weight of total child care consumption in the utility derived from the first period consumption[26], and $\mu$ is the fraction of income consumed in the first period of the agent. The problem of the representative agent can be summarized by the following Lagrangian

$$\begin{aligned} L = & \left[\left(n_t c_{1t}\right)^{\theta}\left(c_{2yt}\right)^{1-\theta}\right]^{\mu}\left[c_{2ot+1}\right]^{1-\mu} \\ & +\bar{\lambda}\left\{w_t l + \frac{\left(1-\lambda_{t-1}\right)}{n_{t-1}}\left(1+r_t\right)s_{t-1} - \left[n_t p_t c_{1t} + c_{2yt} + \frac{c_{2ot+1}}{(1+r_{t+1})}\right]\right\} \end{aligned} \tag{3.7}$$

The maximization of the Lagrangian gives the first order necessary conditions to be satisfied by an optimal solution. Consequently, optimal consumptions of children, adults and elderly should satisfy the following first order conditions:

$$c_{2yt} = \left(\frac{1-\theta}{\theta}\right) n_t c_{1t} p_t \tag{3.8}$$

---

[26] $\theta$ can be interpreted as a constant that governs the degree of altruism of parents towards their children.

$$c_{2ot+1}=\left(\frac{1-\mu}{\mu\theta}\right)\left(1+r_{t+1}\right)\left(n_t c_{1t}\right)p_t \tag{3.9}$$

Plugging (3.8) and (3.9) into the budget constraint given in (3.6) we can derive consumption demands per child, per worker and per old as

$$c_{1t}=\mu\theta\frac{\left(w_t l+b_{t-1}\right)}{n_t}\frac{1}{p_t} \tag{3.10}$$

$$c_{2yt}=\left(1-\theta\right)\mu\left(w_t l+b_{t-1}\right) \tag{3.11}$$

$$c_{2ot+1}=\left(1-\mu\right)\left(1+r_{t+1}\right)\left(w_t l+b_{t-1}\right) \tag{3.12}$$

where

$$b_{t-1}=\frac{\left(1-\lambda_{t-1}\right)}{n_{t-1}}s_{t-1}\left(1+r_t\right) \tag{3.13}$$

is equal to the accidental bequest received by each adult living at time $t$. One of the merits of adding the children to the model as a third generation is the fact that the model can capture the positive (negative) scale effect of declining (increasing) fertility rate on the well being of children. This effect is apart and independent from the impact of fertility through the price mechanism. Its source is the $n_t$ term in the denominator of (3.10) whose increase (decline) lowers (raises) the amount of good 1 consumed by each child. As discussed in the next chapter, this scale effect of fertility is also determining for the gap between child and adult consumption as a measure of intra-family inequality.

Substitution of (3.10), (3.11) and (3.12) in the saving equation (3.4) yields savings of a working adult at time t:

$$s_t=\left(1-\mu\right)\left(w_t l+b_{t-1}\right) \tag{3.14}$$

where $1-\mu$ is the constant saving rate in the economy.

## 3.3 Production Technology and the Problem of Firms

The production of good 1 is assumed to be relatively labor intensive and the production of good 2 is assumed to be relatively capital intensive. The production of these goods is governed by the production function $F_1$ and $F_2$ respectively, where $\frac{dF_i}{dK} > 0$ and $\frac{dF_i}{dL} > 0$ for $i = 1,2$, where $K$ and $L$ are capital and labor inputs. Let's denote the total capital stock and labor used in the two sectors by $K_{it}$ and $L_{it}$ and the total output produced by the two sectors by $X_{it}$ for $i = 1,2$. Then firms in sector 1 and sector 2 solve the following profit maximization problems:

$$
\begin{aligned}
&\underset{L_{1t},K_{1t}}{Max}\ \Pi_{1t} = p_t X_{1t} - w_t L_{1t} - r_t K_{1t} \\
&\text{and} \\
&\underset{L_{2t},K_{2t}}{Max}\ \Pi_{2t} = X_{2t} - w_t L_{2t} - r_t K_{2t} \\
&\text{where } L_{1t} + L_{2t} = N_t l = N_t(l_{1t} + l_{2t}) \text{ and} \\
&K_{1t} + K_{2t} = K_t
\end{aligned}
\tag{3.15}
$$

These problems can be expressed in per worker terms as

$$
\begin{aligned}
&Max\ \pi_{1t} = p_t x_{1t} - w_t l_{1t} - r_t k_{1t} \\
&\text{and} \\
&Max\ \pi_{2t} = x_{2t} - w_t l_{2t} - r_t k_{2t} \\
&\text{where } l_{1t} + l_{2t} = l \text{ and} \\
&k_{1t} + k_{2t} = k_t
\end{aligned}
\tag{3.16}
$$

where $\pi_{1t}$ and $\pi_{2t}$ are per worker profits, and $k_{it}, x_{it}$ for $i = 1,2$, are sectoral per worker capital stocks and outputs respectively. We assume a constant return to scale production technology of Cobb-Douglas form. Production technology uses capital

($K$) and labor ($L$) as inputs so that the total outputs in sector one and two are represented as follows:

$$X_{1t} = F_1(K_{1t}, L_{1t}) = AK_{1t}^{\ \alpha} L_{1t}^{\ 1-\alpha} \tag{3.17}$$

and

$$X_{2t} = F_2(K_{2t}, L_{2t}) = AK_{2t}^{\ \beta} L_{2t}^{\ 1-\beta} \tag{3.18}$$

where $1 > \beta > \alpha > 0$, because good 2 was assumed to be relatively capital intensive. For simplicity, we assume that constant shift parameter $A$ in the production function is the same across sectors. Since the production functions are CRS, we can express everything in per worker terms and get

$$x_{1t} = Ak_{1t}^{\ \alpha} l_{1t}^{\ 1-\alpha} \tag{3.19}$$

and

$$x_{2t} = Ak_{2t}^{\ \beta} l_{2t}^{\ 1-\beta} \tag{3.20}$$

The firms in sectors one and two solve their profit maximization problem given in (3.16), and we derive the following first order conditions:

$$r_t = A\alpha \left( \frac{k_{1t}}{l_{1t}} \right)^{\alpha-1} p_t = A\beta \left( \frac{k_{2t}}{l_{2t}} \right)^{\beta-1} \tag{3.21}$$

and

$$w_t = A(1-\alpha) \left( \frac{k_{1t}}{l_{1t}} \right)^{\alpha} p_t = A(1-\beta) \left( \frac{k_{2t}}{l_{2t}} \right)^{\beta} \tag{3.22}$$

where clearance of capital and labor market requires that

$$k_t = k_{1t} + k_{2t} \tag{3.23}$$

and

$$l = l_{1t} + l_{2t} \tag{3.24}$$

Dividing (3.21) and (3.22) side by side, we get the following expressions for sectoral per worker capital stocks:

$$k_{1t} = \delta l_{1t} p_t^{\frac{1}{\beta-\alpha}} \tag{3.25}$$

$$k_{2t} = \varepsilon l_{2t} p_t^{\frac{1}{\beta-\alpha}} \tag{3.26}$$

where

$$\varepsilon = \left(\frac{\alpha}{\beta}\right)^{\frac{\alpha}{\beta-\alpha}} \left(\frac{1-\beta}{1-\alpha}\right)^{\frac{\alpha-1}{\beta-\alpha}} \tag{3.27}$$

and

$$\delta = \left(\frac{\alpha}{\beta}\right)^{\frac{\beta}{\beta-\alpha}} \left(\frac{1-\beta}{1-\alpha}\right)^{\frac{\beta-1}{\beta-\alpha}} \tag{3.28}$$

Using (3.23) and (3.24),

$$k_{1t} = -\frac{\delta}{\varepsilon-\delta} k_t + \frac{\delta\varepsilon l}{\varepsilon-\delta} p_t^{\frac{1}{\beta-\alpha}} \tag{3.29}$$

$$k_{2t} = \frac{\varepsilon}{\varepsilon-\delta} k_t - \frac{\delta\varepsilon l}{\varepsilon-\delta} p_t^{\frac{1}{\beta-\alpha}} \tag{3.30}$$

Using (3.25)-(3.26) and (3.29)-(3.30), sectoral demands for per worker effective labors can be written as

$$l_{1t} = -\frac{1}{\varepsilon-\delta} k_t p_t^{\frac{1}{\alpha-\beta}} + \frac{\varepsilon l}{\varepsilon-\delta} \tag{3.31}$$

$$l_{2t} = \frac{1}{\varepsilon-\delta} k_t p_t^{\frac{1}{\alpha-\beta}} - \frac{\delta l}{\varepsilon-\delta} \tag{3.32}$$

Using (3.21)-(3.22) and (3.25)-(3.26) the rental rate and wage rate can be expressed in terms of the relative price of the first good as:

$$r_t = A\alpha\delta^{\alpha-1} p_t^{\frac{\beta-1}{\beta-\alpha}} = A\beta\varepsilon^{\beta-1} p_t^{\frac{\beta-1}{\beta-\alpha}} \tag{3.33}$$

$$w_t = A(1-\alpha)\delta^{\alpha} p_t^{\frac{\beta}{\beta-\alpha}} = A(1-\beta)\varepsilon^{\beta} p_t^{\frac{\beta}{\beta-\alpha}} \tag{3.34}$$

Once again using (3.25)-(3.26), per worker sectoral outputs can be written in terms of effective labor inputs and relative price as:

$$x_{1t} = A\delta^{\alpha} l_{1t} p_t^{\frac{\alpha}{\beta-\alpha}} \tag{3.35}$$

$$x_{2t} = A\varepsilon^{\beta} l_{2t} p_t^{\frac{\beta}{\beta-\alpha}} \tag{3.36}$$

Then, if we plug (3.31) into (3.35) and (3.32) into (3.36), we obtain that:

$$x_{1t} = A\left[\frac{\varepsilon l}{\varepsilon-\delta} - \frac{1}{\varepsilon-\delta} k_t p_t^{\frac{1}{\alpha-\beta}}\right]\delta^{\alpha} p_t^{\frac{\alpha}{\beta-\alpha}} \tag{3.37}$$

$$x_{2t} = A\left[\frac{1}{\varepsilon-\delta} k_t p_t^{\frac{1}{\alpha-\beta}} - \frac{\delta l}{\varepsilon-\delta}\right]\varepsilon^{\beta} p_t^{\frac{\beta}{\beta-\alpha}} \tag{3.38}$$

## 3.4 Inter-temporal Equilibrium in the Autarkic Economy

The capital and goods markets clearance at each period implies the following conditions:

$$N_{t+1}k_{t+1} = N_t s_t \tag{3.39}$$

and

$$x_{1t} = n_t c_{1t} \tag{3.40}$$

$$\begin{aligned} N_t(x_{2t}+k_t) &= N_t c_{2yt} + O_t c_{2ot} + N_{t+1}k_{t+1} \Rightarrow \\ x_{2t}+k_t &= c_{2yt} + E_t c_{2ot} + Y_t k_{t+1} \end{aligned} \tag{3.41}$$

Equation (3.39) states that per worker savings should be equal to per worker capital stock at each period, while (3.40) and (3.41) represent the goods market clearance conditions for the first and the second goods respectively.

A perfect foresight equilibrium is a sequence $\{k_t, p_t\}_{t=0}^{\infty}$ that clears the goods' market at every period t while satisfying the dynamics of the capital stock at time t+1. First, in reference to (3.39) and (3.14), the law of motion for per worker capital stock can be written as

$$k_{t+1} = \frac{(1-\mu)(w_t l + b_{t-1})}{n_t} \tag{3.42}$$

The goods market clearance conditions for the two goods were given in (3.40) and (3.41). By Walras' Law, we can concentrate on the market clearance condition for the first good which is given by

$$x_{1t} = n_t c_{1t}$$

Using (3.13) and (3.14), and doing a recursive substitution, we can express the per worker accidental bequest per worker received at time $t$ by

$$b_{t-1} = \left(\prod_{k=1}^{t-1} \frac{1-\lambda_k}{n_k}(1+r_{k+1})\right)(1-\mu)^{t-1} b_0 + \sum_{j=1}^{t-1} w_j l (1-\mu)^{t-j} \left(\prod_{k=j}^{t-1} \frac{1-\lambda_k}{n_k}(1+r_{k+1})\right) \tag{3.43}$$

It is assumed that workers of the initial period do not receive accidental bequests from deceased parents (i.e., we assume $s_1 = 0$ meaning that old agents living at the initial period did not save for their old age consumption) implying that $b_0 = 0$. Then equation (3.43) reduces to

$$b_{t-1} = \sum_{j=1}^{t-1} w_j l (1-\mu)^{t-j} \left[\prod_{k=j}^{t-1} \frac{1-\lambda_k}{n_k}(1+r_{k+1})\right] \tag{3.44}$$

We can substitute $w_j$ and $r_{k+1}$ as given in (3.34) and (3.33) in the above equation to write the bequests in terms of the relative prices:

$$b_{t-1} = A\left(1-\alpha\right)\delta^{\alpha} l \sum_{j=1}^{t-1} p_j^{\frac{\beta}{\beta-\alpha}} \left(1-\mu\right)^{t-j} \left[\prod_{k=j}^{t-1} \frac{1-\lambda_k}{n_k}\left(1+A\alpha\delta^{\alpha-1} p_{k+1}^{\frac{\beta-1}{\beta-\alpha}}\right)\right]$$

We can further express the product term on the right-hand side in open form as follows:

$$\prod_{k=j}^{t-1} \frac{1-\lambda_k}{n_k}\left(1+A\alpha\delta^{\alpha-1} p_{k+1}^{\frac{\beta-1}{\beta-\alpha}}\right) = \psi_{t-1}\prod_{k=j}^{t-1}\left(1+\phi p_{k+1}^{\frac{\beta-1}{\beta-\alpha}}\right) \tag{3.45}$$

where

$$\psi_{t-1} = \prod_{k=j}^{t-1} \frac{1-\lambda_k}{n_k} \tag{3.46}$$

$$\phi = A\alpha\delta^{\alpha-1} \tag{3.47}$$

Consequently, $b_{t-1}$ can be reduced to

$$b_{t-1} = A\left(1-\alpha\right)\delta^{\alpha} l \left\{\sum_{j=1}^{t-1} p_j^{\frac{\beta}{\beta-\alpha}} \left(1-\mu\right)^{t-j} \psi_{t-1}\prod_{k=j}^{t-1}\left(1+\phi p_{k+1}^{\frac{\beta-1}{\beta-\alpha}}\right)\right\} \tag{3.48}$$

Now, using (3.10), (3.34), (3.48) and the market clearance condition for the first good $x_{1t} = n_t c_{1t}$ , we can write per worker output of sector 1 as

$$x_{1t} = \Lambda A\delta^{\alpha}\left\{p_t^{\frac{\alpha}{\beta-\alpha}} + p_t^{-1}\sum_{j=1}^{t-1}\left[p_j^{\frac{\beta}{\beta-\alpha}} \left(1-\mu\right)^{t-j} \psi_{t-1}\prod_{k=j}^{t-1}\left(1+\phi p_{k+1}^{\frac{\beta-1}{\beta-\alpha}}\right)\right]\right\} \tag{3.49}$$

where

$$\Lambda = \mu\theta\left(1-\alpha\right)l \tag{3.50}$$

In order to write all terms containing $p_t$ in explicit form, we can take out the product term containing $p_t$ from the product $\prod_{k=j}^{t-1}\left(1+\phi p_{k+1}{}^{\frac{\beta-1}{\beta-\alpha}}\right)$ outside the summation and write

$$x_{1t} = \Lambda A\delta^{\alpha} p_t{}^{\frac{\alpha}{\beta-\alpha}} + \Lambda A\delta^{\alpha}\left\{p_t{}^{-1}\left(1+\phi p_t{}^{\frac{\beta-1}{\beta-\alpha}}\right)\sum_{j=1}^{t-1}\left[p_j{}^{\frac{\beta}{\beta-\alpha}}\left(1-\mu\right)^{t-j}\psi_{t-1}\prod_{k=j}^{t-2}\left(1+\phi p_{k+1}{}^{\frac{\beta-1}{\beta-\alpha}}\right)\right]\right\} \tag{3.51}$$

Using the definition of $x_{1t}$ in (3.35) and solving for effective labor per worker used in sector 1 we can write

$$l_{1t} = \Lambda\left\{1+p_t{}^{\frac{\beta}{\alpha-\beta}}\left(1+\phi p_t{}^{\frac{\beta-1}{\beta-\alpha}}\right)\left[\sum_{j=1}^{t-1}p_j{}^{\frac{\beta}{\beta-\alpha}}\left(1-\mu\right)^{t-j}\psi_{t-1}\prod_{k=j}^{t-2}\left(1+\phi p_{k+1}{}^{\frac{\beta-1}{\beta-\alpha}}\right)\right]\right\} \tag{3.52}$$

Now, let

$$\Omega_{t-1} = \sum_{j=1}^{t-1}p_j{}^{\frac{\beta}{\beta-\alpha}}\left(1-\mu\right)^{t-j}\psi_{t-1}\prod_{k=j}^{t-2}\left(1+\phi p_{k+1}{}^{\frac{\beta-1}{\beta-\alpha}}\right) \tag{3.53}$$

Substituting (3.31) in the above equation we can solve for $k_t$ in terms of contemporaneous and lagged values of relative price of the first good as

$$k_t = \varepsilon l p_t{}^{\frac{1}{\beta-\alpha}} + \Lambda\left(\delta-\varepsilon\right)\left\{1+p_t{}^{\frac{\beta}{\alpha-\beta}}\left(1+\phi p_t{}^{\frac{\beta-1}{\beta-\alpha}}\right)\Omega_{t-1}\right\} \tag{3.54}$$

Rewriting (3.42) for time t in terms of lagged relative prices by using (3.34) and (3.48) we have

$$k_t = \frac{\Phi_2 l}{n_{t-1}}\left\{p_{t-1}{}^{\frac{\beta}{\beta-\alpha}} + \left(1+\phi p_{t-1}{}^{\frac{\beta-1}{\beta-\alpha}}\right)\Omega_{t-2}\right\} \tag{3.55}$$

where $\Phi_2 = A\left(1-\mu\right)\left(1-\alpha\right)\delta^{\alpha}$.

Equating (3.54) with (3.55), we attain a highly nonlinear difference equation of order t that gives us the law of motion for relative prices as follows:

$$\varepsilon l p_t^{\frac{1}{\beta-\alpha}} + \Lambda(\delta-\varepsilon)\left\{1+p_t^{\frac{\beta}{\alpha-\beta}}\left(1+\phi p_t^{\frac{\beta-1}{\beta-\alpha}}\right)\Omega_{t-1}\right\} = p_{t-1}^{\frac{\beta}{\beta-\alpha}}\frac{\Phi_2 l}{n_{t-1}}\left\{1+p_{t-1}^{\frac{\beta}{\alpha-\beta}}\left(1+\phi p_{t-1}^{\frac{\beta-1}{\beta-\alpha}}\right)\Omega_{t-2}\right\}$$

(3.56)

A dynamic equilibrium of the autarky economy can be characterized by constant fertility and survival rates, $n_s$ and $\lambda_s$, a steady state price ratio $p_s$ and per worker capital stock $k_s$ where,

**i.** $n_{t+1} = n_t = n_s$, $\lambda_{t+1} = \lambda_t = \lambda_s$, $p_{t+1} = p_t = p_s$ and $k_{t+1} = k_t = k_s$ for all $t \geq s$, where $s$ is the index for the period at which the steady state is reached.

**ii.** $p_s$ and $k_s$ are such that the markets for good 1 and good 2 clear, i.e., (3.40) and (3.41) are satisfied.

**iii.** $p_s$ and $k_s$ obey the law of motion for the capital stock, i.e., equation (3.39) is satisfied.

Unlike the models in Sayan (2005) and Jelassi (2004), closed form solutions for the steady state of this autarky economy is not analytically attainable, if mortality is assumed to be positive. The presence of bequests, as a byproduct of positive mortality, serves as a channel for intergenerational wealth transfers. Therefore, the expression for bequests in (3.48) becomes a function of the whole price sequence starting from the initial price ratio. Thus the goods and capital market clearance conditions result in an n$^{th}$ order nonlinear difference equation as given in (3.56) whose analytical solution is not attainable. Existence of the dynamic equilibrium (steady state) defined above will be demonstrated in the next chapter by using numerical solutions of the model. The next section will present the analytical solution of the steady state values for the model variables under the assumption that adult mortality is equal to zero. Then a comparative static exercise will be carried out to provide some insight into the effects of fertility on the steady state values.

## 3.5 Dynamic Equilibrium under Full Certainty of Survival into Old Age

If adult mortality is assumed to be zero, i.e., $\lambda_t = 1 \ \forall t \geq 1$, then it is easy to derive the closed form solutions for the steady state values of capital stock and the price ratio. Under no mortality assumption, bequests will be ruled out and agents will be consuming out of their own wage income only. Since the probability of death for adults is zero, bequests will also be zero, i.e., $b_{t-1} = 0 \ \forall t \geq 2$.

Using the market clearance condition for good 1 in (3.40), and substituting (3.10) and (3.34) into this equation, we can express per worker output of sector 1 as

$$x_{1t} = A\mu\theta\left(1-\alpha\right)\delta^{\alpha} p_t^{\frac{\alpha}{\beta-\alpha}} l \tag{3.57}$$

Equating the expression for $x_{1t}$ in (3.35) to the one above, we have

$$l_{1t} = \mu\theta\left(1-\alpha\right) l$$

Then equating the expression in (3.31) for effective labor employed in sector 1 to the one above we can solve for per worker capital stock as

$$k_t = \left[\varepsilon\left(1-\mu\theta\left(1-\alpha\right)\right) + \delta\mu\theta\left(1-\alpha\right)\right] l p_t^{\frac{1}{\beta-\alpha}} \tag{3.58}$$

Using (3.34) and (3.42) we had

$$k_{t+1} = A\left(1-\mu\right)\left(1-\alpha\right)\delta^{\alpha} l \frac{1}{n_t} p_t^{\frac{\beta}{\beta-\alpha}} \tag{3.59}$$

Writing (3.58) for time t+1 and equating it with (3.59) we get

$$p_{t+1}^{\frac{1}{\beta-\alpha}} = \frac{A\left(1-\mu\right)\left(1-\alpha\right)\delta^{\alpha}}{n_t\left[\varepsilon\left(1-\mu\theta\left(1-\alpha\right)\right)+\delta\mu\theta\left(1-\alpha\right)\right]} p_t^{\frac{\beta}{\beta-\alpha}} \tag{3.60}$$

Since at the steady state $p_{t+1} = p_t = p_s$, using (3.60), the steady state price ratio is expressed as

$$p_s = \left(n_s \frac{\Phi_1}{\Phi_2}\right)^{\frac{\beta-\alpha}{\beta-1}} \tag{3.61}$$

where

$$\Phi_1 = \left[\varepsilon\left(1-\mu\theta\left(1-\alpha\right)\right)+\delta\mu\theta\left(1-\alpha\right)\right] \tag{3.62}$$

and as before,

$$\Phi_2 = A\left(1-\mu\right)\left(1-\alpha\right)\delta^{\alpha} \tag{3.63}$$

Using (3.58) we can alternatively solve for $p_t$ in terms of $k_t$ to get

$$p_t = \left(\frac{k_t}{\Phi_1 l}\right)^{\beta-\alpha} \tag{3.64}$$

Then plugging (3.64) in (3.59) the law of motion for the capital stock can be written as

$$k_{t+1} = \left(\frac{\Phi_2 l}{n_t}\right)\left(\frac{k_t}{\Phi_1 l}\right)^{\beta} \tag{3.65}$$

At the steady state, $k_{t+1} = k_t = k_s$. Then using (3.65) the steady state capital stock is written as

$$k_s = \left(n_s \frac{\Phi_1^{\beta}}{\Phi_2}\right)^{\frac{1}{\beta-1}} l = \left(n_s \frac{\Phi_1}{\Phi_2}\right)^{\frac{1}{\beta-1}} \Phi_1 l \tag{3.66}$$

**Proposition 1:** *Under the assumption of zero adult mortality, the autarky equilibrium price ratio* $p_s$ *given in (3.61) for this perfect foresight overlapping generations general equilibrium model with constant returns to scale technology and convex preferences exists and is unique if* $\alpha \in (0,1)$, $\beta \in (0,1)$, $\alpha \neq \beta$, $\mu \in [0,1)$, $\theta \in [0,1]$, $A > 0$ *and* $n_s > 0$.

**Proof:** From (3.61) we had $p_s = \left( n_s \frac{\Phi_1}{\Phi_2} \right)^{\frac{\beta-\alpha}{\beta-1}} = \left( \frac{\Phi_2}{n_s \Phi_1} \right)^{\frac{\beta-\alpha}{1-\beta}}$. Note that $\alpha \neq \beta$. First, assume that $\beta > \alpha$. Then $\frac{\beta-\alpha}{1-\beta} > 0$ since $\beta < 1$. Therefore, the existence of $p_s$ is ensured if $\frac{\Phi_2}{n_s \Phi_1}$ is well-defined, i.e., $n_s \Phi_1 = n_s \left[ \varepsilon \left( 1 - \mu\theta(1-\alpha) \right) + \delta\mu\theta(1-\alpha) \right]$ should be non-zero. Since $\alpha \in (0,1)$ and $\beta \in (0,1)$, $\varepsilon = \left( \frac{\alpha}{\beta} \right)^{\frac{\alpha}{\beta-\alpha}} \left( \frac{1-\beta}{1-\alpha} \right)^{\frac{\alpha-1}{\beta-\alpha}} > 0$ and $\delta = \left( \frac{\alpha}{\beta} \right)^{\frac{\beta}{\beta-\alpha}} \left( \frac{1-\beta}{1-\alpha} \right)^{\frac{\beta-1}{\beta-\alpha}} > 0$. Moreover, $0 < \left[ 1 - \mu\theta(1-\alpha) \right] \leq 1$ and $0 \leq \mu\theta(1-\alpha) < 1$ follow from the fact that $\mu \in [0,1)$, $\theta \in [0,1]$ and $\alpha \in (0,1)$. We thus conclude that $\varepsilon \left[ 1 - \mu\theta(1-\alpha) \right] > 0$ and $\delta\mu\theta(1-\alpha) \geq 0$, which together with $n_s > 0$ imply that $n_s \Phi_1 > 0$. Now, assume that $\alpha > \beta$. Then, $\frac{\beta-\alpha}{\beta-1} > 0$, and $p_s = \left( n_s \frac{\Phi_1}{\Phi_2} \right)^{\frac{\beta-\alpha}{\beta-1}}$ is well-defined if and only if $\Phi_2 \neq 0$. It is easy to observe that $\Phi_2 = A(1-\mu)(1-\alpha)\delta^{\alpha} > 0$, since $\delta > 0$, $\mu \in [0,1)$, $\alpha < 1$ and $A > 0$. So, $p_s$ is well defined for $\alpha \neq \beta$. Finally, note that $p_s > 0$ automatically follows from $\Phi_1 > 0$, $\Phi_2 > 0$ and $n_s > 0$. Thus, the long-run equilibrium price ratio $p_s$ exists. Uniqueness of $p_s$ is trivial from equation (3.61). □

In fact, if we comply with the restriction that $\beta > \alpha$, then the share of first period consumption $\mu$ can also become unity, since, as can be noted from the above proof, under this restriction, setting $\mu = 1$ does not make the expression for the steady state price ratio undefined. However, since $\mu = 1$ would mean to ignore the old generation, it is not in line with the essence of a three-generation OLG model.

Having established the existence and uniqueness of $p_s$, the existence and uniqueness of $k_s$ follow from the fact that $k_s = \Phi_1 l p_s^{\frac{1}{\beta-\alpha}}$, which is strictly positive when $l > 0$. Now it is easy to derive the steady state solutions for other endogenous variables using $p_s$, $k_s$ and the necessary conditions that follow from profit and utility maximization problems. The sector specific per worker capital stocks at the steady state are given by

$$k_{1s} = \frac{\delta l}{\delta - \varepsilon}\left(n_s \frac{\Phi_1}{\Phi_2}\right)^{\frac{1}{\beta-1}}\left(\Phi_1 - \varepsilon\right) \tag{3.67}$$

and

$$k_{2s} = \frac{\varepsilon l}{\varepsilon - \delta}\left(n_s \frac{\Phi_1}{\Phi_2}\right)^{\frac{1}{\beta-1}}\left(\Phi_1 - \delta\right) \tag{3.68}$$

Effective labors per worker employed in each sector at the steady state are

$$l_{1s} = \frac{1}{\delta - \varepsilon} l\left(\Phi_1 - \varepsilon\right) \tag{3.69}$$

$$l_{2s} = \frac{1}{\varepsilon - \delta} l\left(\Phi_1 - \delta\right) \tag{3.70}$$

Steady state rental and wage rates are given as

$$r_s = A\alpha\delta^{\alpha-1}\left(n_s\frac{\Phi_1}{\Phi_2}\right) = A\beta\varepsilon^{\beta-1}\left(n_s\frac{\Phi_1}{\Phi_2}\right) \tag{3.71}$$

and

$$w_s = A(1-\alpha)\delta^{\alpha}\left(n_s\frac{\Phi_1}{\Phi_2}\right)^{\frac{\beta}{\beta-1}} = A(1-\beta)\varepsilon^{\beta}\left(n_s\frac{\Phi_1}{\Phi_2}\right)^{\frac{\beta}{\beta-1}} \tag{3.72}$$

Sectoral outputs per worker are

$$x_{1s} = A\frac{1}{\delta-\varepsilon}l(\Phi_1-\varepsilon)\delta^{\alpha}\left(n_s\frac{\Phi_1}{\Phi_2}\right)^{\frac{\alpha}{\beta-1}} \tag{3.73}$$

and

$$x_{2s} = A\frac{1}{\varepsilon-\delta}l(\Phi_1-\delta)\varepsilon^{\beta}\left(n_s\frac{\Phi_1}{\Phi_2}\right)^{\frac{\beta}{\beta-1}} \tag{3.74}$$

Steady state per child consumption of good 1 is given as

$$c_{1s} = A\mu\theta(1-\alpha)l\delta^{\alpha}n_s^{\frac{1+\alpha-\beta}{\beta-1}}\left(\frac{\Phi_1}{\Phi_2}\right)^{\frac{\alpha}{\beta-1}} \tag{3.75}$$

while the steady state per worker and per old consumptions of good 2 can be expressed by

$$c_{2ys} = A\mu(1-\theta)(1-\alpha)l\delta^{\alpha}\left(n_s\frac{\Phi_1}{\Phi_2}\right)^{\frac{\beta}{\beta-1}} \tag{3.76}$$

and

$$c_{2os} = \left[\Phi_2 + A\alpha\delta^{\alpha-1}n_s\Phi_1\right]l\left(n_s\frac{\Phi_1}{\Phi_2}\right)^{\frac{\beta}{\beta-1}} \tag{3.77}$$

Finally, the steady state level of per worker savings can be expressed by

$$s_s = \Phi_2 l \left( n_s \frac{\Phi_1}{\Phi_2} \right)^{\frac{\beta}{\beta - 1}} \tag{3.78}$$

## 3.6 The Effect of the Fertility Rate on Long-run Model Variables

Having derived the closed-form solutions for the model variables under zero mortality assumption, the impact of a rise or fall in fertility on the equilibrium magnitudes can be analytically demonstrated through the following corollaries.

**Corollary 1:** *The long-run relative price $p_s$ of good 1 is decreasing in the rate of steady state fertility $n_s$ if the production of good 1 is relatively labor intensive, i.e., $\beta > \alpha$; and $p_s$ is increasing in $n_s$ if the production of good 1 is relatively capital intensive, i.e., $\alpha > \beta$.*

**Proof:** Differentiating the term for the steady state price ratio given in (3.61) with respect to $n_s$ it is seen that

$$\frac{\partial p_s}{\partial n_s} = \left( \frac{\beta - \alpha}{\beta - 1} \right) \left( \frac{\Phi_1}{\Phi_2} \right)^{\frac{\beta - \alpha}{\beta - 1}} n_s^{\frac{1-\alpha}{\beta - 1}}$$

Note that $n_s > 0$. Given that the conditions in Proposition 1 hold, $\beta - 1 < 0$, $\Phi_1 > 0$ and $\Phi_2 > 0$. Hence, $\frac{\partial p_s}{\partial n_s} \begin{cases} < 0 \text{ if } \beta > \alpha \\ > 0 \text{ if } \beta < \alpha \end{cases}$. □

Other things constant, Corollary 1 states that in the long-run, in a country with higher fertility rate, the relative price of a labor intensive good is lower than in a country with lower fertility rate. Then since the steady state population growth rate is equal to $n_s - 1$, as the population growth rate increases the country gains a comparative advantage in the production of labor intensive goods, and as the population growth rate falls the country gains a comparative advantage in the production of capital intensive goods. This result is in line with the results in Sayan (2005) and Jelassi (2004), and it implies that differences in fertility rates can induce trade between two otherwise identical countries. Trade implications of the basic model and the model that will be introduced in Chapter 5 will be investigated in detail in Chapter 6 using a dynamic HO framework.

**Corollary 2:** *The long-run wage rate and capital per worker are decreasing in steady state fertility rate* $n_s$*, while the long-run rental rate is increasing in* $n_s$.

**Proof:** The derivatives of the equilibrium wage rate and capital per worker with respect to fertility rate are

$$\frac{\partial w_s}{\partial n_s} = \frac{A\beta(1-\alpha)\delta^{\alpha}}{\beta-1}\left(\frac{\Phi_1}{\Phi_2}\right)^{\frac{\beta}{\beta-1}} n_s^{\frac{1}{\beta-1}}$$

and

$$\frac{\partial k_s}{\partial n_s} = \frac{1}{\beta-1}\left(\frac{\Phi_1{}^{\beta}}{\Phi_2}\right)^{\frac{1}{\beta-1}} n_s^{\frac{2-\beta}{\beta-1}}$$

Since $\beta-1<0$ and other terms are all positive, $\frac{\partial k_s}{\partial n_s}<0$ and $\frac{\partial w_s}{\partial n_s}<0$. As for the rental rate,

$$\frac{\partial r_s}{\partial n_s} = A\alpha\delta^{\alpha-1}\left(\frac{\Phi_1}{\Phi_2}\right)>0. \ \square$$

Corollary 2 states that in a country with higher fertility rate, real wages and per worker capital will be lower, and rental rates will be higher, other things being constant. This is intuitive because as the capital-labor ratio decreases, the marginal productivity of labor, which is equal to real wage under perfect competition, is expected to fall, while the marginal productivity of capital will increase. In other words, labor is cheaper and capital is more expensive in high population growth and hence younger countries, because these countries are relatively labor abundant and capital scarce. Knowing that capital per worker is decreasing in $n_s$, it is not surprising to see that per worker savings is also decreasing in fertility rate as verified by:

$$\frac{\partial s_s}{\partial n_s} = \frac{A(1-\mu)\beta(1-\alpha)\delta^{\alpha} l}{\beta-1}\left(\frac{\Phi_1}{\Phi_2}\right)^{\frac{\beta}{\beta-1}} n_s^{\frac{1}{\beta-1}} < 0 .$$

**Corollary 3:** *Equilibrium per worker capital stocks employed in both sectors move in the same direction in response to a change in steady state fertility. In particular,* $\frac{\partial k_{1s}}{\partial n_s} \leq 0$ *and* $\frac{\partial k_{2s}}{\partial n_s} < 0$. *However, the equilibrium allocation of sectoral effective labors between the two sectors are not affected by any change in fertility, i.e.,* $\frac{\partial l_{is}}{\partial n_s} = 0$ *for* $i = 1,2$.

**Proof:** The derivative of per worker capital in sector 1 is equal to

$$\frac{\partial k_{1s}}{\partial n_s} = \frac{\delta(\Phi_1-\varepsilon) l}{(\delta-\varepsilon)(\beta-1)}\left(\frac{\Phi_1}{\Phi_2}\right)^{\frac{1}{\beta-1}} \frac{1}{n_s}$$

Since $\delta-\varepsilon<0$ and $\beta-1<0$, the sign of the derivative is equal to the sign of $\Phi_1-\varepsilon=\mu\theta(1-\alpha)(\delta-\varepsilon)$. By the conditions given in Proposition 1, $\mu\in[0,1)$ which implies that $\Phi_1-\varepsilon\leq 0$. Then $\frac{\partial k_{1s}}{\partial n_s}\leq 0$. Similarly,

$$\frac{\partial k_{2s}}{\partial n_s} = \frac{\varepsilon(\Phi_1-\delta) l}{(\varepsilon-\delta)(\beta-1)}\left(\frac{\Phi_1}{\Phi_2}\right)^{\frac{1}{\beta-1}} \frac{1}{n_s} < 0$$

Since $\Phi_1-\delta=(\varepsilon-\delta)(1-\mu\theta(1-\alpha))>0$ and $(\varepsilon-\delta)(\beta-1)<0$.

Finally, it is clear from equations (3.69) and (3.70) that $\frac{\partial l_{is}}{\partial n_s} = 0$ for $i = 1, 2$. □

So, Corollary 3 states that in equilibrium, both sectors reduce their per worker capital utilization in response to an increase in steady state fertility, but sectoral allocation of total effective labor is independent of any change in $n_s$. From Corollary 3, it follows that sectoral outputs per worker should also be decreasing in the level of steady state fertility, i.e., $\frac{\partial x_{1s}}{\partial n_s} \leq 0$ and $\frac{\partial x_{2s}}{\partial n_s} < 0$.

**Corollary 4:** *The equilibrium amount of consumption per child $c_{1s}$ and consumption per adult $c_{2ys}$ are decreasing in steady state fertility, whereas consumption per old $c_{2os}$ may increase or decrease in response to a rise in fertility.*

**Proof:** From the derivatives of $c_{1s}$ and $c_{2ys}$ with respect to $n_s$, it can be noted that

$$\frac{\partial c_{1s}}{\partial n_s} = \underbrace{A\mu\theta(1-\alpha)\delta^{\alpha} l}_{\geq 0} \underbrace{\left(\frac{1+\alpha-\beta}{\beta-1}\right)}_{<0} \underbrace{\left(\frac{\Phi_1}{\Phi_2}\right)^{\frac{\alpha}{\beta-1}} n_s^{\frac{2(1-\beta)+\alpha}{\beta-1}}}_{>0} \leq 0$$

and

$$\frac{\partial c_{2ys}}{\partial n_s} = \underbrace{A\mu(1-\theta)(1-\alpha)\delta^{\alpha} l}_{\geq 0} \underbrace{\left(\frac{\beta}{\beta-1}\right)}_{<0} \underbrace{\left(\frac{\Phi_1}{\Phi_2}\right)^{\frac{\beta}{\beta-1}} n_s^{\frac{1}{\beta-1}}}_{>0} \leq 0$$

Finally,

$$\frac{\partial c_{2os}}{\partial n_s} = \left(\frac{\beta}{\beta-1}\right)\Phi_2 l \left(\frac{\Phi_1}{\Phi_2}\right)^{\frac{\beta}{\beta-1}} n_s^{\frac{1}{\beta-1}} \left[1 + A\alpha\delta^{\alpha-1} n_s \frac{\Phi_1}{\Phi_2}\right] + A\alpha\delta^{\alpha-1}\Phi_1 l \left(n_s \frac{\Phi_1}{\Phi_2}\right)^{\frac{\beta}{\beta-1}}$$

The impact of a change in steady state level of fertility on the steady state level of old age consumption is ambiguous, since the first term in the above expression is negative and the second term in the sum is positive. After doing necessary simplifications, in particular we have $\frac{\partial c_{2os}}{\partial n_s}\begin{cases} \geq 0 \text{ if } & n_s \geq (1-\mu)\delta\left(\frac{\beta}{\alpha}\right)\left(\frac{1-\alpha}{1-\beta}\right)\frac{1}{\Phi_1} \\ < 0 \text{ if } & \textit{otherwise} \end{cases}$.

Thus, the direction of the impact depends on the relative magnitudes of the constant parameters of technology and utility function and on the rate of fertility. If the fertility rate is above the threshold level of fertility, then a rise in fertility affects the amount of consumption by old positively, in contrast to the amounts of consumptions per child and adult. □

In summary, Corollary 4 suggests that higher fertility makes each child and working adult worse off in the long-run, whereas the old agents may become better off or worse off depending on the model parameters and on the level of $n_s$.

**Corollary 5:** *The equilibrium level of life-time utility of the representative agent is decreasing in the steady state fertility, if* $\frac{\partial c_{2os}}{\partial n_s} < 0$.

**Proof:** At the steady state, the life-time utility of a representative agent is given by

$$U_s = \left[(n_s c_{1s})^{\theta} c_{2ys}{}^{1-\theta}\right]^{\mu} c_{2os}{}^{1-\mu}$$

Then the derivative of the life-time utility with respect to steady state fertility $n_s$ is written as

$$\frac{\partial U_s}{\partial n_s} = \mu\left[\frac{(n_s c_{1s})^{\theta} c_{2ys}{}^{1-\theta}}{c_{2os}}\right]^{\mu-1}\left[c_{2ys}{}^{1-\theta}\left(\theta n_s{}^{\theta-1} c_{1s}{}^{\theta} + \theta n_s{}^{\theta} c_{1s}{}^{\theta-1}\frac{\partial c_{1s}}{\partial n_s}\right) + (1-\theta)(n_s c_{1s})^{\theta}\frac{\partial c_{2ys}}{\partial n_s}\right]$$
$$+(1-\mu)\left[\frac{(n_s c_{1s})^{\theta} c_{2ys}{}^{1-\theta}}{c_{2os}}\right]^{\mu}\frac{\partial c_{2os}}{\partial n_s}$$

Assume that $n_s \geq (1-\mu)\delta\left(\frac{\beta}{\alpha}\right)\left(\frac{1-\alpha}{1-\beta}\right)\frac{1}{\Phi_1}$ so that by Corollary 4, $\frac{\partial c_{2os}}{\partial n_s} \geq 0$.

## 3.7 Summary

This chapter introduced the basic OLG model with three-generations, two factors and two sectors as benchmark model to analyze the separate effects of fertility and mortality on the model variables. The model differs from those in Sayan (2005) and Jelassi (2004) basically on three fronts. First, this model involves not only exogenously given fertility rates, but also positive mortality rates. Second, this model involves three generations (children, adults and old) as opposed to two.[27] Lastly, this model assumes that each generation consumes only one type of good. In particular, children consume only the labor-intensive good, while adults and old consume only the capital-intensive good.

As a necessity that arises from the introduction of mortality, the model also includes bequests as a channel of intergenerational wealth transfer. In this basic model, although agents' lifetime budget constraint is affected by the introduction of mortality and bequests, the agents are assumed to be unaware of the probability of death they actually face. In other words they cannot anticipate $\lambda_t$. Having shown that the model has no analytical solution under positive mortality rate, the closed form

[27] It should be underlined that children constitute a generation in a narrow sense, because they do not solve an optimization problem until they become adults in the next period. The amount they consume is decided and purchased by their parents according to an altruistic bond introduced in the utility function. The introduction of children to the model has the purpose of being able to define a child dependency ratio besides the old age dependency ratio. Moreover, this formulation has the merit of capturing the scale effect on consumption per child. A rise (decline) in fertility results in an extra decline (rise) in the amount of consumption per child in addition to the regular income effect. In this sense, adding children to the model as a third generation enables us to analyze the effects of fertility on the gap between consumption per child and per head consumption by other generations.

solutions of the equilibrium level of the model variables are attained by assuming away the mortality rate. Under this assumption, the existence and uniqueness of the steady state price ratio and per worker capital have been demonstrated and a comparative static exercise has been carried out to see the effect of fertility on the steady state of the economy. Consequently, the outcomes of the comparative static exercise with respect to steady state fertility can be summarized by the following findings:

$\frac{\partial k_s}{\partial n_s} < 0$, $\frac{\partial p_s}{\partial n_s} \begin{cases} < 0 \text{ if } \beta > \alpha \\ > 0 \text{ if } \beta < \alpha \end{cases}$, $\frac{\partial k_{1s}}{\partial n_s} \leq 0$, $\frac{\partial k_{2s}}{\partial n_s} < 0$, $\frac{\partial r_s}{\partial n_s} > 0$, $\frac{\partial w_s}{\partial n_s} < 0$, $\frac{\partial s_s}{\partial n_s} < 0$, $\frac{\partial x_{1s}}{\partial n_s} \leq 0$,

$\frac{\partial x_{2s}}{\partial n_s} < 0$, $\frac{\partial c_{1s}}{\partial n_s} < 0$ and $\frac{\partial c_{2ys}}{\partial n_s} \leq 0$.

These results imply that fertility differences among countries are theoretically sufficient to create a basis for trade, because differences in fertility rates create differences in relative labor and capital endowments which, according to the static HO trade model, is the reason for mutually beneficial trade. However, as Sayan (2005) reports, under a dynamic HO setting, trade induced by population growth differentials may not be mutually beneficial. Whether this finding is verified by this basic model and the other model that will be introduced in Chapter 5 will be discussed in Chapter 6.

# CHAPTER 4:

# Numeric Solution of the Basic Model with Positive Mortality and Simulation Experiments

As discussed in Chapter 3, the autarky model with adult mortality and bequests does not lend itself for closed form solutions of long-run equilibrium price ratio and capital stock. The main focus of this chapter is on the impact of shocks to mortality and the way these shocks are transmitted across generations. To this end, the model will be numerically solved under various demographic scenarios, and the rest of the chapter is devoted to the presentation and discussion of results from a rich set of demographic simulations.

## 4.1 Numeric Solution and Simulation Methodology for the Basic Autarkic Economy

The numeric solution of the basic model described in Chapter 3 was obtained using the MATLAB code written for this purpose. The transition paths of the endogenous variables to their respective steady states and the steady state solutions themselves have been calculated using a grid search method. The price ratio is assumed to have reached its steady state when the absolute change in the price ratio drops below $10^{-6}$. Unless stated otherwise, the numeric solutions of the models presented in the rest of the thesis are carried out under the set of common parameter values in Table 4.1.

**Table 4.1: Technology and Preference Parameters for the Numeric Solution of the Basic Model**

| $\alpha$ | $\beta$ | $\theta$ | $\mu$ | $l$ | $A$ | $b_1$ [28] | $C_1, N_1, O_1$ |
|---|---|---|---|---|---|---|---|
| 0.3 | 0.5 | 0.4 | 0.8 | 1 | 1 | 0 | 1, 1, 1 |

As discussed earlier $\alpha$ and $\beta$ are the share parameters of capital in the production of the good 1 and good 2 respectively. Since good 2 is the relatively capital-intensive good, $\alpha$ is chosen to be greater than $\beta$. The particular values for these production parameters are borrowed from Jelassi (2004), which reports estimates of capital shares in output ranging from 26% to more than 60% in a study conducted by Chenery (1986). Also in line with Jelassi (2004), $\mu$ and $\theta$ are assigned the values given in Table 4.1. The selection of $\mu$ is consistent with the unweighted average of the private saving rate of 20% in industrial countries, since $1-\mu$ is equal to the constant saving rate in the economy as given in (3.14). The selection of $\theta$ together with $\mu = 0.8$ implies that the fraction of wage income spent on child care consumption is 32% and the fraction of wage income spent on adult consumption is 48%. The effective labor endowment $l$ of each worker is normalized to unity so that $l_1$ and $l_2$ stand for the fraction of total labor time devoted to the production of good 1 and good 2

[28] This variable is the level of bequests received by each member of the initial adult generation.

respectively. The shift parameter $A$ in the production function is the same for both goods. It shows the total factor productivity in the production and is set to be equal to one. The results are qualitatively robust to the selection of $A$, because changing this parameter only scales the magnitude of the variables up or down. The initial per worker bequests is set to equal to zero to simply the calculation of initial relative price that is necessary for the numeric solution. Similarly, initial sizes of child, adult and old populations are all equal to one which is the minimum number required to initialize the evolution of the population.

Except where it is noted otherwise, all simulation experiments are carried out using sectoral capital stocks set equal to the corresponding steady state values obtained with the steady state birth and death rates in that experiment. The initial price ratio $p_1$ is calculated using the market clearance condition for good 1. At this stage, it is assumed that adults do not know that they may not survive to their retirement period. In other words, the probability of death is not incorporated in the utility function. The reason for the adoption of this rather unrealistic assumption is to draw on the contrast between the qualitative nature of the results under this assumption and under the more realistic model, where changing mortality affects intertemporal resource allocation of agents by altering the saving rates. This latter case will be discussed in the next chapter. In the following sections, economic implications of demographic shifts will be analyzed using the basic model introduced in Chapter 3, under the assumption of 'no mortality anticipation'.

The main motivations behind the population experiments in this chapter are **(i)** to see the separate effect of a change in each individual population parameter on our

OLG economy in the long-run, **(ii)** to see the combined short and medium run effects of different population shocks.

## 4.2 Long-run Implications of Demographic Change

To accomplish the first task mentioned above, a baseline scenario is defined to serve as a benchmark for evaluating other scenarios. This baseline scenario, called Base 1.1, where assumes that adult mortality is zero, i.e. $\lambda_t = 1,\ \forall t$, and the birth rate or fertility is constant at $n_t = n = 1.65,\ \forall t$ throughout the whole model horizon. This will result in a growing population with an old-age dependency ratio of $E = \frac{1}{n} < 1$ and a child dependency ratio of $Y = n > 1$.

The first set of simulation scenarios aims to capture the behavior of the steady state values when there is a shift from no mortality to positive mortality among adults and a shift from high fertility to lower fertility. Hence, the scenarios analyzed are the following:

i. **Simulation 1.1:** The first scenario assumes $1 - \lambda_t = 1 - \lambda = 0.6,\ \forall t$, which is calculated to give us the projected old age dependency ratio for the whole world, while $n_t = 1.65 > 1,\ \forall t$ as in Base 1.1.

ii. **Simulation 1.2:** The second scenario assumes a lower fertility rate of $n = 1$ than $n = 1.65$ in Base 1.1, while taking $\lambda_t = 1,\ \forall t$ as in Base 1.1.

### 4.2.1 The Baseline Scenario and Stability of the Equilibrium

Figure 4.1 summarizes the transition of the main variables in the economy to their respective steady states under the Base scenario 1.1. We can analyze the dynamics that leads the economy to its steady state by considering the interconnections among the goods, capital and labor markets. Since all markets are perfectly competitive at each period, consumers and producers take prices as given. Figure 4.1 shows that relative price of good 1, $p$, steadily increases until around period 10. Real wages increase and the rental rates fall. Hence, the income of agents steadily rises, stimulating increasing consumption and savings. It can be observed from the figure that per capita child, adult and old consumptions rise. $c_{2o}$ is well above $c_1$ because old agents spend their whole savings together with the accumulated interest on good 2. Although $c_1$ is slightly above $c_{2y}$, the opposite is also possible if the fertility rate is increased sufficiently. Equations (3.75) and (3.76) indicate that the gap widens in favor of per child consumption if $\beta - \alpha$ or $\theta$ increases, and the gap narrows down if the steady state fertility rises.

On the production side, producers facing higher demand for the outputs of both sectors are willing to produce more and increase their absolute demand for capital and labor. Since the total relative demand for the capital intensive good (good 2) is higher than for the labor intensive good (good 1), per worker production of the former is above that of the latter, explaining the fact that $k_{2t} > k_{1t}$ as seen in the first panel of Figure 4.1. Bequests, $b$, are zero since the adult mortality is assumed to be zero in Base 1.1. Therefore this scenario corresponds to the numeric solution of the model given in Section 3.5., where equation (3.59) shows that per worker capital stock at time $t+1$ is increasing in the relative price of good 1 at time $t$ as long as $\beta > \alpha$, i.e. as long as production of good 2 is relatively capital intensive. This explains why capital stock per worker steadily increases until the steady state.

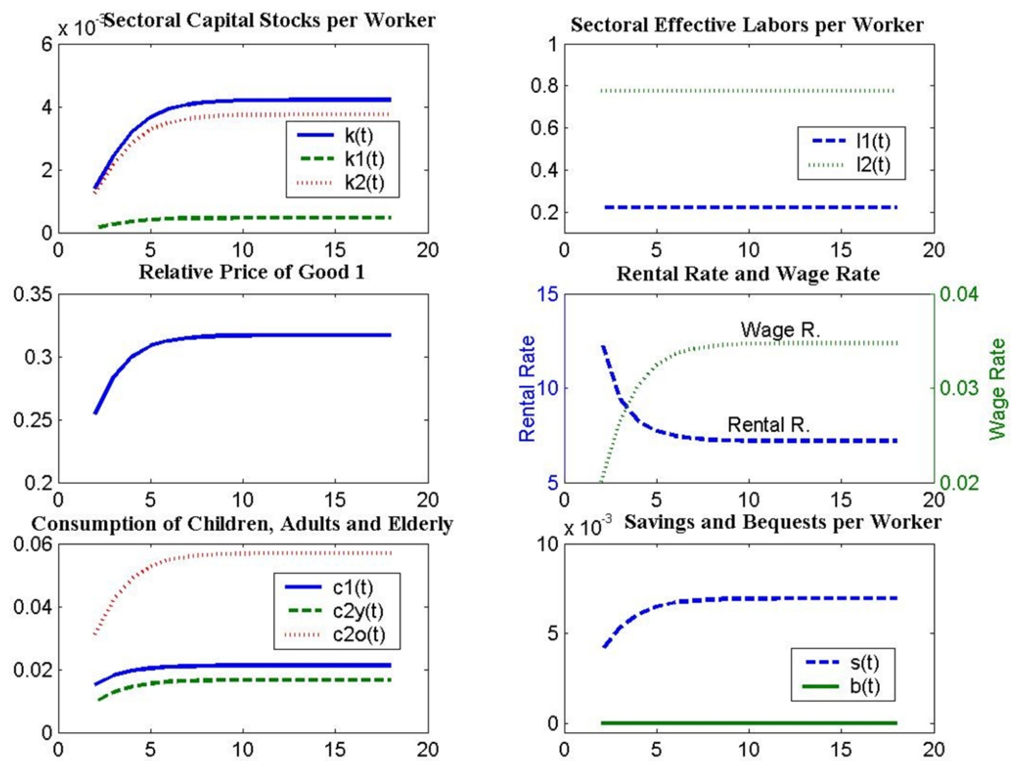


**Figure 4.1: Results of Scenario Base 1.1 ($\lambda = 1$, $n = 1.65$)**

Since the production technology is constant returns to scale (CRS) for each producer, the elasticity of substitution between labor and capital is equal to one for each sector. Hence, it follows that at a given level of production a one percentage point increase in the rate of technical substitution of labor for capital (RTS), where $RTS = MP_L / MP_K = w/r$, causes an exactly one percentage point increase in the sectoral capital-to-labor ratio for both producers, $\frac{K_i}{L_i}$. Using this equality and expressing everything in per worker terms, it can be concluded that at any point in time $\frac{(1+g_{k1})}{(1+g_{l1})}=\frac{(1+g_{k2})}{(1+g_{l2})}$, where $g_{ki}$ and $g_{li}$ show the growth rates of per worker capital stock and effective labor employed in sector $i$. With the initial parameter values given in Table 4.1, and given the constant population growth rate of 0.65, the sectoral allocation of one unit of effective labor remains constant as in Figure 4.1. Although sector 1 is the relatively labor intensive sector, the share of effective labor employed in sector 2 is well above the share employed in sector 1. This follows from the complementarities between capital and labor. In order to achieve a higher level of per worker output compared to sector 1's output, optimality requires sector 2 to employ not only a higher level of $k_2$ but also a higher share from the unit labor supply, since the marginal productivity of capital is increasing in the level of labor used. Given that $g_{l1}$ and $g_{l2}$ are equal to zero, the above equality implies an equal rate of growth for $k_1$ and $k_2$ along the steady state path, i.e., $g_{k1t}=g_{k2t}$=$\mathrm{g_t}$ , $\forall t\geq 1$, where $g_{1kt}$ and $g_{k2t}$ denote the growth rate of capital per worker employed in sector 1 and sector 2 respectively. But then, since $k_t=k_{1t}+k_{2t}$, from $k_{t+1}=k_{1t+1}+k_{2t+1}=(k_{1t}+k_{2t})(1+g_t)=k_t(1+g_t)$ we have $g_{k1t}=g_{k2t}=g_{kt}=g_t$ at each period $t$ throughout the model horizon. This result is also verified by calculating $g_{k1}, g_{k2}$ and $g_k$ for each $t$ until the steady state using the numerical solution.

The phase diagrams for $k_t$ and $p_t$ given in Figure 4.2 and Figure 4.3 show that the steady state equilibrium characterized by the $(k_s, p_s)$ pair is globally asymptotically stable.

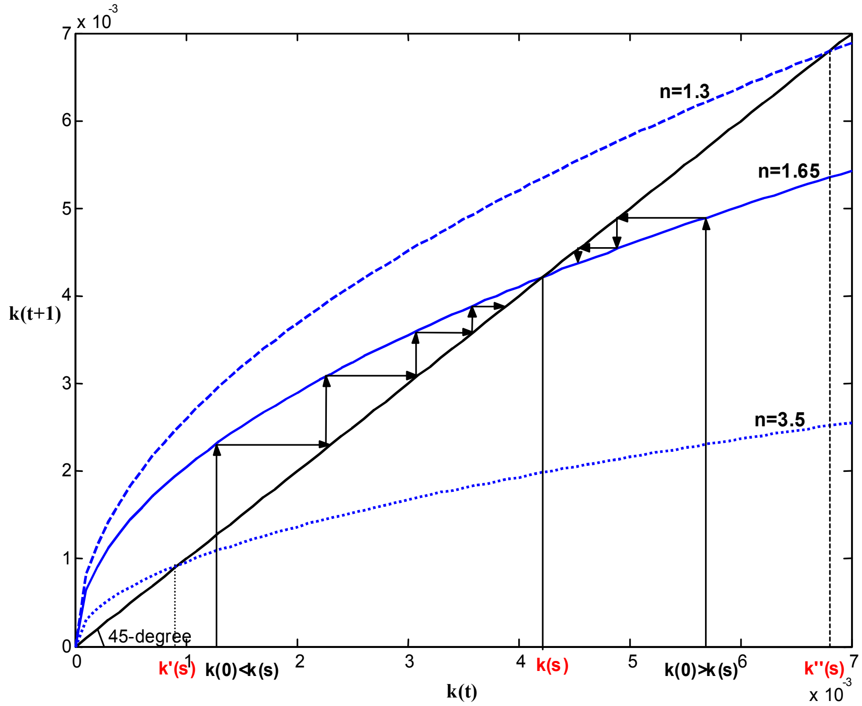


**Figure 4.2: The Phase Diagram of the Capital Stock Dynamics (No Adult Mortality)**

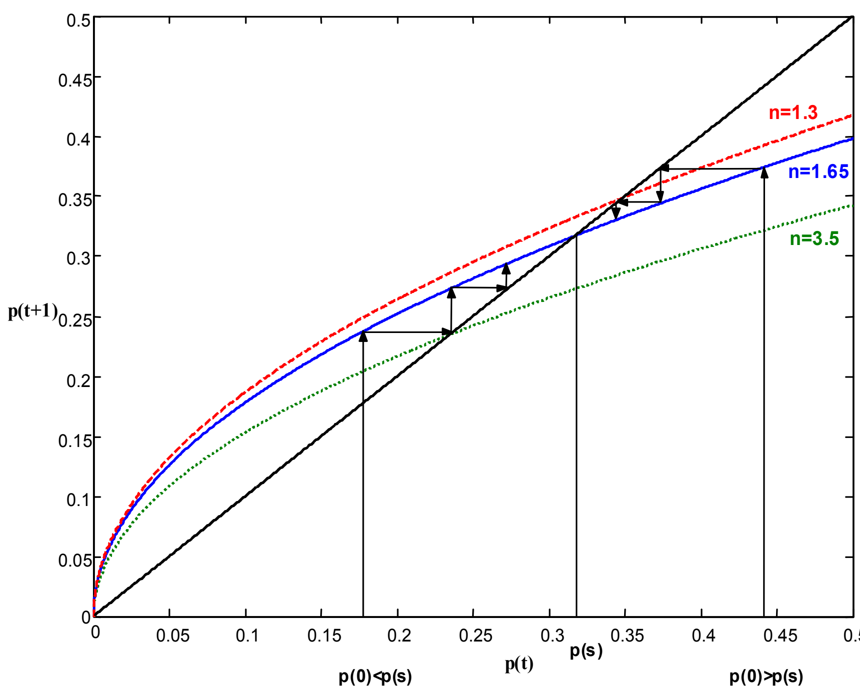


**Figure 4.3: The Phase Diagram for the Price Dynamics (No Adult Mortality)**

The first diagram shows that for a given level of fertility, $n$, the capital stock converges to the unique steady state regardless of the initial level of capital stock chosen. If the economy starts below the steady state level of per worker capital stock, per worker capital rises monotonically until the steady state is reached.Conversely, if the initial capital stock is above the steady state level, $k$ monotonically falls to reach $k_s$. The same is true for the price dynamics shown in the second diagram. Whether the relative price in the economy starts from above or below its steady state value, it will end up at the unique equilibrium price ratio $p_s$.

### 4.2.2 Increasing the Adult Mortality

Simulation 1.1 introduces an adult mortality rate of 0.6, which is selected arbitrarily only to better visualize the effects of increasing adult mortality. As a result, deceased agents start to leave unintentional bequests to the next generation. Figure 4.4 summarizes the effect of increasing mortality on the steady state capital stock, prices, sectoral labor allocation and wage and rental rates. 'Low' and 'High' in the legends refer to the level of mortality. The time paths denoted by 'Low' correspond to Base 1.1. where $n = 1.65$ and $\lambda = 1$, i.e., where mortality is lower than in Sim. 1.1. The time paths denoted by 'High' correspond to Sim. 1.1. where $n = 1.65$, but $\lambda = 0.4$, i.e., where mortality is higher than in Base 1.1. The plots on the upper- and lower-left panels of Figure 4.4 suggest that an increase in the steady state adult mortality raises the level per worker capital stock and the relative price of the labor-intensive good. The intuition behind this observation is as follows: Given a constant fertility rate, when the adult mortality rate increases, a higher proportion of the total wealth will be transferred to next period's workers in the form of increasing bequests per worker.

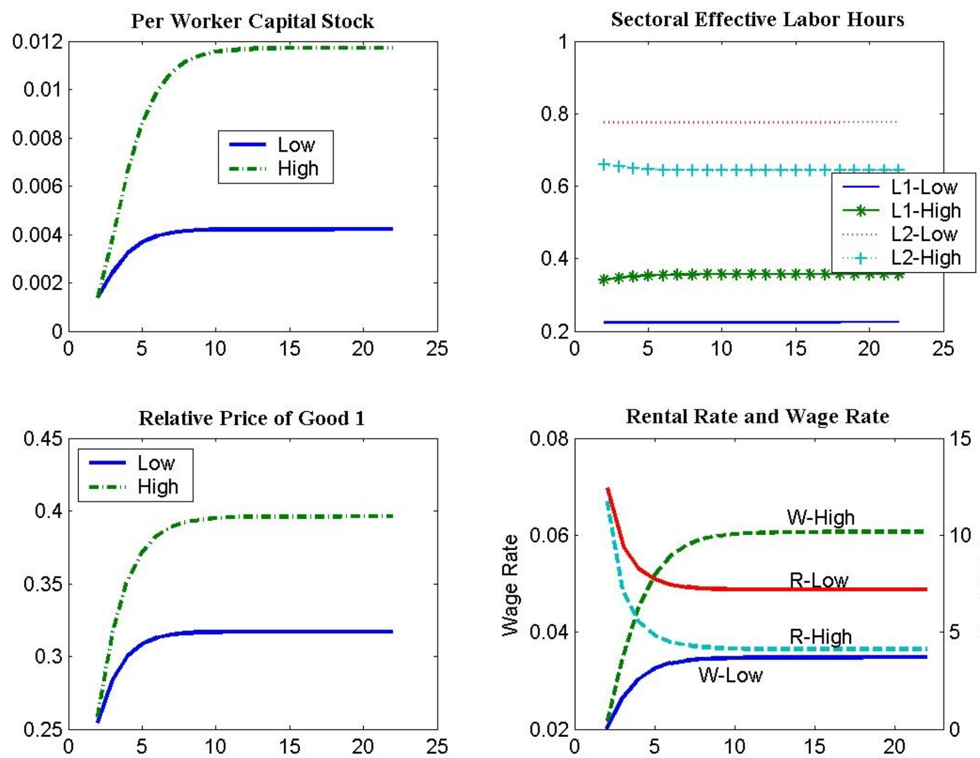


**Figure 4.4: Results of Scenario Sim.1.1.**

This positive wealth effect raises savings per worker as well as the amount of consumption. Thus capital per worker, i.e., capital-labor ratio, increases meaning that the economy becomes more capital abundant than before. Hence, rental rates fall and wages rise where the latter results in a positive income effect and the former reduces the return on savings and puts a negative pressure on bequests. Consistent with the changing factor prices, the relative price of labor-intensive good rises. In line with the intergenerational wealth transfer induced by increasing adult mortality, wealth is diverted away from old consumption towards the next generation's wealth. Thus, the relative demand for good 2 declines because a smaller fraction of adults demands good 2 for old age consumption. This in turn means a rise in the relative demand for good 1 and hence a higher $p$ in consistence with the explanation given above. In line with this rise in $p$, the labor share of sector 1 increases and the labor share of sector 2 falls, as evident from the upper-right panel of Figure 4.4. The lower-right panel shows that in line with the fall in the relative price, the steady state wage rate is higher and the steady state rental rate is lower than the benchmark case.

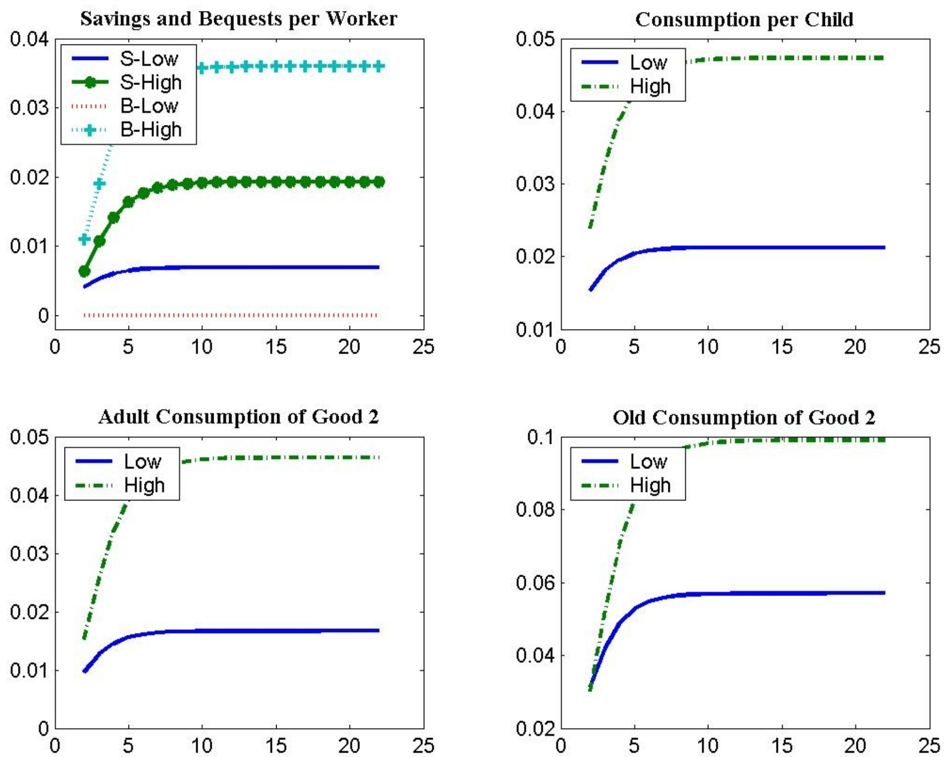


**Figure 4.5: Results of Scenario Sim. 1.1. (Cont'd)**

As argued in the previous paragraph, lower survival rate means lower elderly dependency ratio, and hence relatively lower demand for good 2, which is consumed only by adults and elderly. This means that relative demand for good 1 and hence $p$ will be higher at the steady state.

Finally, steady state allocation of sectoral labor is also altered due to increasing mortality. Although sector 2 still employs a much larger share of total labor, there is a downward shift in this share due to the factor substitution of labor for capital in the production of consumption-invesment good (good 2). This is simply because of the decline in the price of capital $r$ relative to the price of labor $w$. Figure 4.5 summarizes the long-run impact of increasing mortality on savings, bequests and consumption. In response to higher $p$, steady state rental rate falls and wage rate rises, raising the income of workers further. Savings soar and hence capital accumulation intensifies, resulting in higher output in each sector. An increase in mortality from 0% to 60% raises the level of bequests left by each generation from 0 to around slightly below 0.04. Thus, the wealth of the next generation increases. Indeed, this wealth effect stimulates higher consumption and saving as shown in Figure 4.5. Due to the presence of bequests, some portion of the returns to workers' total savings is diverted away from old-age consumption towards child and adult consumption and capital accumulation. Since the agents are assumed to be unaware of their survival probability, they do not take this information into account and do not lower their savings in the face of a positive mortality rate. This is a crucial and implausible assumption, which may and indeed does change the qualitative nature of the results as will be seen in Chapter 5. This issue will be addressed in the next section by relaxing the assumption that death rates cannot be foreseen. An interesting implication of the model with positive mortality is that the intergenerational wealth transfer from old to adults lowers the gap between the level of child consumption and adult consumption. From a numerical comparison of the consumption by adults and children it can be verified that the gap between $c_{1t}$ and $c_{2y}$ is indeed much smaller in the case of high mortality. This effect is also robust to the selection of utility parameter $\theta$. The reason lies in the substitution effect between child consumption and adult consumption in the first period of the agents. Although the levels of both types of consumption ascend to a higher steady state level due to the wealth effect, a substitution occurs at the same time from $c_1$ to $c_{2y}$ due to the declining price of the latter, i.e., due to increasing $p$ .

### 4.2.3 Decreasing the Fertility Rate

The next simulation, Sim.1.2. involves a lower birth rate $n = 1$ and hence a lower young age dependency ratio compared to Base 1.1. This Figure 4.6 and Figure 4.7 compare the time paths of the crucial model variables under high fertility (Base 1.1) and low fertility (Sim.1.2) regimes. The closed form solutions in Section 3.5 of the previous chapter suggest that lower fertility raises the steady state levels of capital stock, hence sectoral outputs, the relative price of the labor intensive good and the wage rate, while lowering the rental rate, compared to the benchmark case involving higher fertility. Moreover, per child consumption of good 1 and adult consumption are also positively affected by this demographic shift. As mentioned before however, the direction of change in the consumption by old agents depends both on the set of constant parameters governing the technology and preferences as well as on the level of steady state birth rate. Figure 4.6 verifies these results. Under lower fertility regime, steady state magnitudes of both relative price and per worker capital stock are above those of higher-fertility regime.

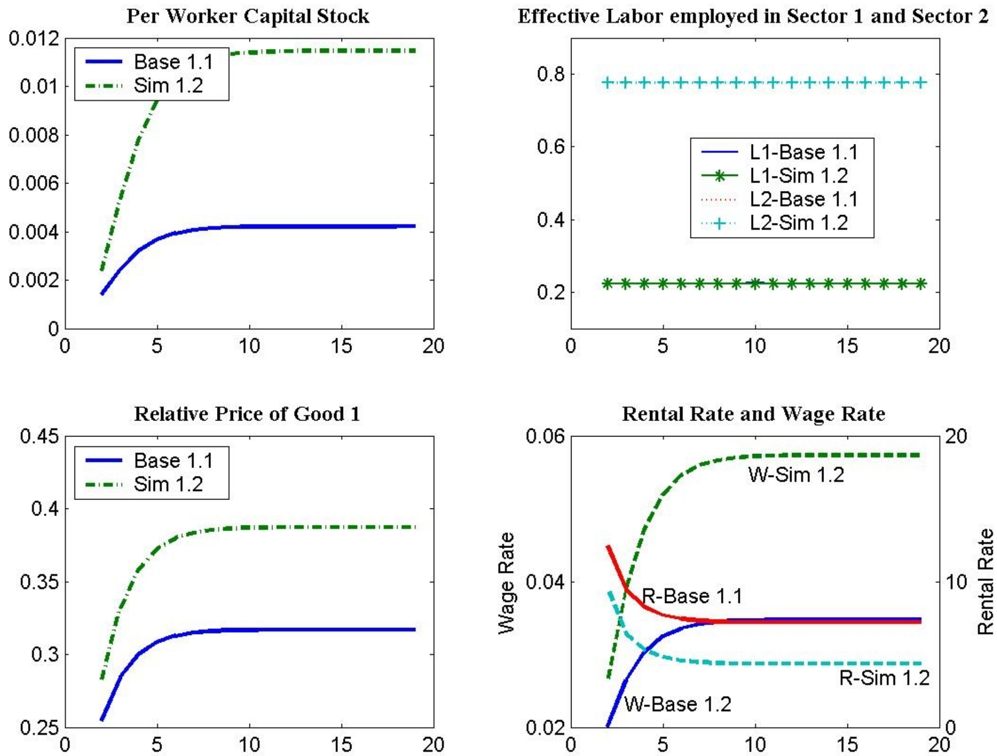


**Figure 4.6: Results of Scenario Sim.1.2.**

In line with the analytical investigation of the effects of fertility in Chapter 3, the plots on the upper-right panel of Figure 4.6 suggest that the change in steady state fertility does not affect the labor allocation, as the time paths of both $l_{1t}$ and $l_{2t}$ under Base 1.1 and Sim.1.2 exactly overlap. From the lower-right panel, it is evident that wages increase and rental rates fall in response to a lower fertility rate. Figure 4.7 reports the effect of lower fertility on savings, bequests and consumption.

The declining child dependency ratio plays an important role in the transition to a higher steady state through the so called positive scale effect. Since the amount of bequests is zero, the consumption level of each child is given by $c_{1t} = \mu\theta \frac{w_t l}{p_t} \frac{1}{n_t}$.

Therefore, other things constant, a lower child dependency ratio $Y_t = n_t$ raises the level of per child consumption.

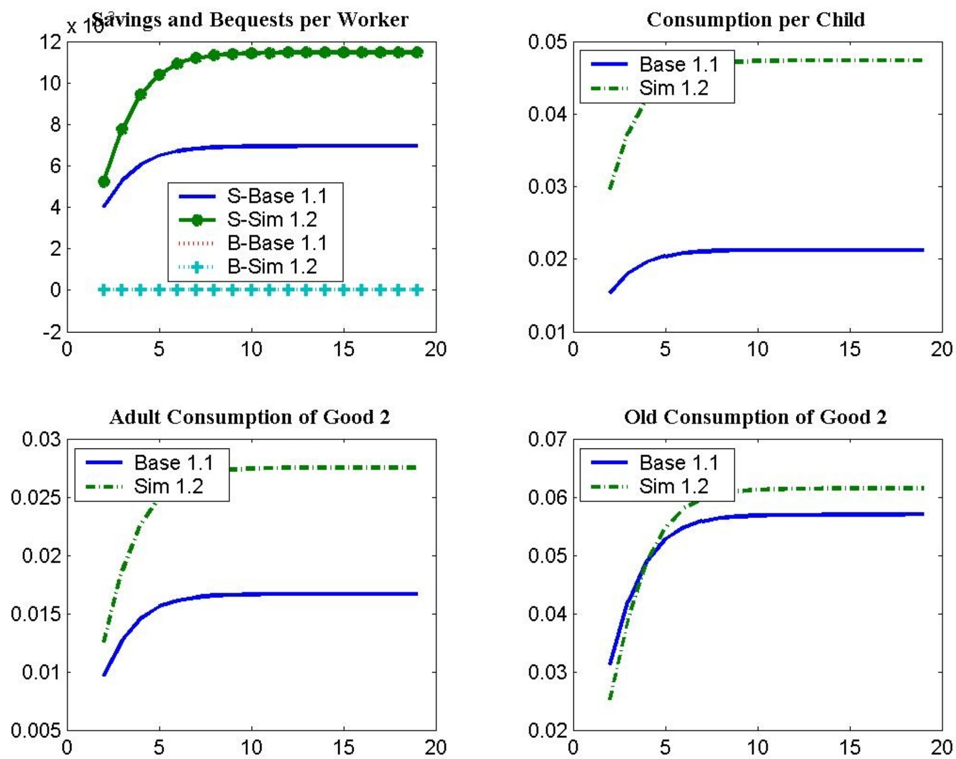


**Figure 4.7: Results of Scenario Sim.1.2. (Cont'd)**

This can be numerically verified by calculating the gap between per head consumptions of good 2 and good 1 at a given point in time.

Compared to Base 1.1., this gap given by $\left(c_{2yt} + c_{2ot}\right) - c_{1t}$ is reduced in Sim.1.2. due to lower child dependency ratio. The level of per worker capital stock is also inversely related to $n_t$ as suggested by the law of motion for capital stock given by $k_{t+1} = \frac{(1-\mu) w_t l}{n_t}$. In other words, the positive scale effect contributes to the price effect by magnifying the positive impact of rising wages and income on the consumption of good 1 and on the capital accumulation. Compared to Base 1.1., the gap between steady state per child consumption and per worker consumption is wider in Sim.1.2. Given that mortality rate is zero, this gap is solely determined by the steady state level of birth rates. Indeed, the closed form solutions for long-run values of $c_1$ and $c_{2y}$, given in equations (3.75) and (3.76) respectively, verify this argument. As the birth rates increase, the gap between the two narrows and vice versa. After a certain threshold, per worker consumption $c_{2y}$ may exceed per child consumption $c_1$. Roughly speaking, when birth rates are below this threshold, the life standards of children living at time $t$ are above their parents in the same period. For the set of parameters given in Table 4.1, child consumption remains above adult consumption unless the average number of children born by each adult within a period exceeds 2.5. Since lower birth rates imply higher welfare, the economic welfare of a country is also positively related to the difference between the life standards of children and their parents at a given point in time. On the other hand, for people living in significantly high fertility countries (where $n$ is above the critical threshold) it is optimal to consume more for themselves than for their children.

## 4.3 Evolving Population Dynamics and Their Implications in the Short-to-Medium Run

The second set of simulations aims to observe the effects of permanent and transitional shocks to population parameters upon the trajectories that lead the economy to the steady state. Therefore, these simulations concentrate on the short-to-medium run impacts of demographic shifts, which are more relevant for policy making. In all scenarios below, shocks are introduced four periods after the initialization of the model, to allow the population dynamics to settle in its intended path. Three different scenarios will be considered here:

i. **Simulation 1.3:** A gradual and permanent decline in death rates, i.e., death rate gradually falls from 0.8 to 0.6, which occurs within four periods (approximately 100 years). Then, $1-\lambda$ remains at its new level. Birth rate is constant at $n=1$.

ii. **Simulation 1.4:** A gradual and permanent decline in birth rates occuring within four periods in a similar fashion to Simulation 1.3. Birth rate falls from 1.65 to 1. On the other hand, death rate is constant at 0.60.

iii. **Simulation 1.5:** The combination of declining fertility rates and declining mortality rates. The timing of fertility and mortality declines are qualitatively in line with the process of demographic transition.

### 4.3.1 Gradually Declining Adult Mortality

Sim.1.3 is designed to analyze the response of economic variables to gradually declining adult mortality rate. Figure 4.8 gives the evolution of birth and death rates described in scenario Sim.1.3., and the corresponding paths of $k_t$ and $p_t$, the consumption levels $c_{1t}$, $c_{2yt}$, $c_{2ot}$, savings and bequests $s_t$, $b_t$ and finally per capita GNP, i.e. per capita real output. As previously discussed, the model is initialized by sectoral capital stocks that are associated with the prospective steady state of the simulation exercise. This allows the system to return to its initial state after each shock, and the pure effect of the demographic change can be isolated more visibly.

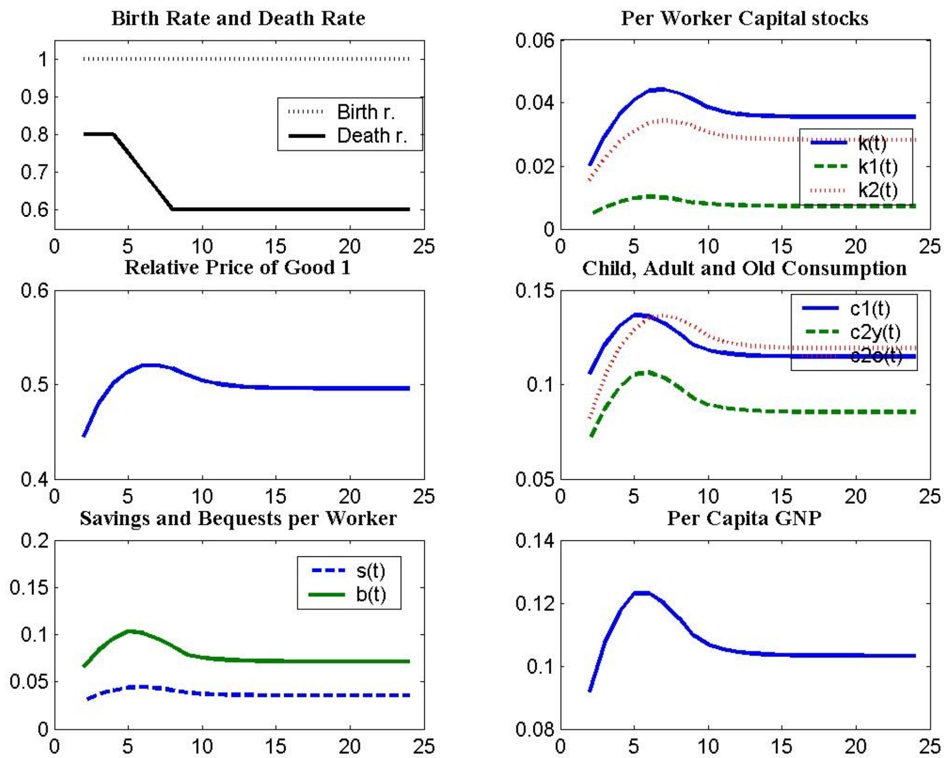


**Figure 4.8: Results for Scenario Sim.1.3.**

While the population moves towards a lower adult mortality, elderly dependency ratio rises. Since old agents consume only from good 2, relative demand for good 2 increases, lowering the relative price of good 1 and hence the wage rate and workers' income. At the same time, given the constant rate of birth, increasing survival rate among elderly pulls down the level of bequests received by each worker, and workers' wealth declines. These two effects (price and bequest effects) lower the consumption and saving of agents. Capital accumulation slows down and consequently per worker capital stock starts to decline. Increasing marginal productivity of capital starts to raise the rental rates. Although this increases the interest paid to bequests, the negative scale effect and the decline in per worker savings on bequests due to higher survival rate overcome the positive rental rate effect. Therefore the net effect on bequests becomes negative. Figure 4.8 confirms this. It suggests that the relative price of good 1 starts to increase until around the sixth period because mortality remains constant at its initial high level until the fourth period. Then, the impact of declining mortality rate kicks in and reverses the rising trend. Hence, the relative price ends up on a lower plateau, when the steady state mortality rate of 0.6 is reached. Along with this descent in $p$, savings and bequests also fall. Consequently, total output and per capita GNP in the economy is negatively affected from this demographic shift.

### 4.3.2 Gradually Declining Fertility Rate

As a counterpart of Sim.1.3, Sim.1.4 involves a gradual decline in the birth rate instead of the death rate. The results are presented in Figure 4.9. For the initial four periods, birth rate remains at a level of 1.65 and then gradually falls down to the replacement rate of one child per adult. When the number of children per adult steadily declines, parents start to spend more for each child due to the scale effect. Since parents get utility from the total amount of child consumption, as $n$ declines, optimality requires $c_{1t}$ to increase assuming that prices and other things remain unchanged. Declining birth rate causes the child dependency ratio to fall, which lowers the relative demand for good 1. Since fertility rate is gradually declining, in each period the economy becomes less labor abundant relative to the previous period when adjusted with respect to the total capital stock. Therefore, the real wage and the relative price of the labor intensive good (good 1) also increase, whereas the rental rate for capital falls (not shown in the figure). Increasing wage rate results in higher income, and higher income means higher consumption and saving.

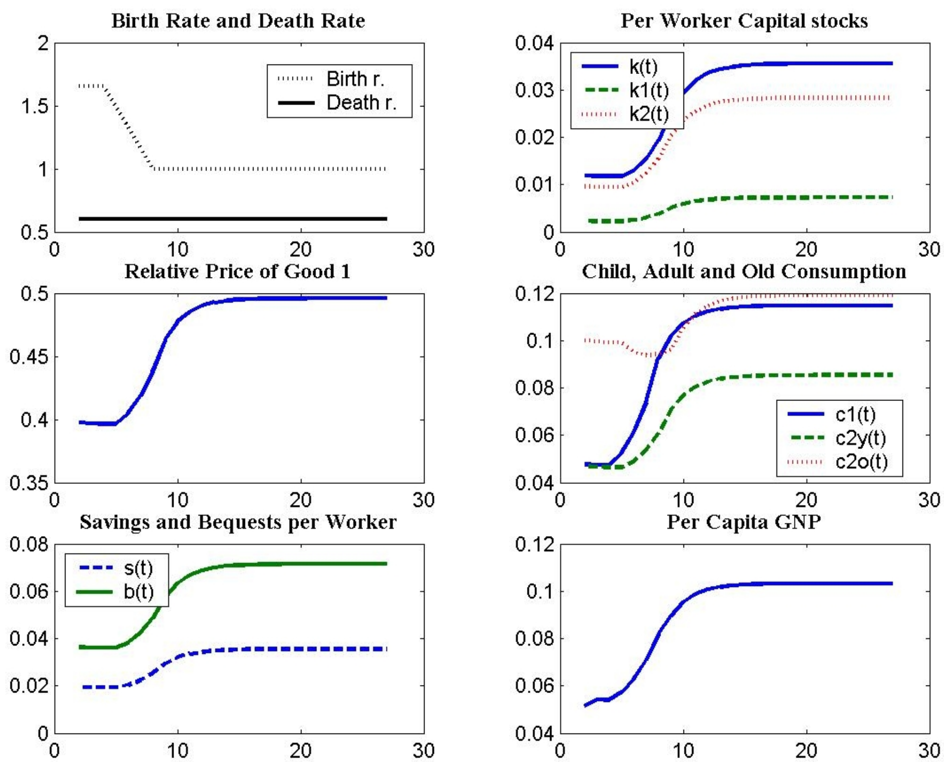


**Figure 4.9: Results of Scenario Sim.1.4.**

Given the constant adult mortality rate, increasing savings raise the level of bequests, while decreasing rental rate partially offsets this positive effect. On the other hand, facing higher $p$ sector 1 is willing to supply more of good 1.

### 4.3.3 A Demographic Transition

In a general sense, the shift from a high mortality and high fertility regime to a low fertility and low mortality regime is called demographic transition. As also pointed out in Bloom, Canning and Malaney (1999), the demographic transition began in most developing countries with a rapid decline in mortality while fertility was roughly stable at a high level. This created a widening gap between mortality and fertility leading to a period of higher population growth until a sharp decline in fertility started. In the model, mortality rate $1-\lambda$ is defined as the adult mortality, and as mentioned before, a decline in child mortality can be accounted for by increasing $n$ accordingly. Since during the demographic transition, the declines in infant and child mortality rates were sharper than the decline in adult mortality, this should be reflected in the evolution of $n$ in the form of an initial boom in fertility in order to obtain a more realistic representation of demographic transition. To this end, the measure of fertility $n$ in the model can be adjusted by letting $n = (\text{infant survival rate}) \times (\text{net reproduction rate})$ and using the UN data and projections on the average infant mortality and net reproduction rate for less developed regions in the periods 1950-1955 and 2045-2050.

The resulting scenario, which is be referred as Sim. 2.1, tries to capture the effects of a demographic transition. As shown in Figure 4.10, the death rate, i.e., the adult mortality, starts to fall at the fourth period from its initial level of one and converge to zero.[29] In the meantime, $n$[30] starts at 1.5 and suddenly jumps to 1.95 in the fifth period. Starting with the sixth period it gradually falls and until reaching the replacement level of one. Convergence of both the birth and death rates to their

[29] This very roughly corresponds to an increase in the life expectancy at birth from 50 to 75 years. These initial and terminal values of adult mortality approximately correspond to the average life expectancies at birth for the less developed regions in the period 1950-1955 and 2045-2050 respectively.
[30] The initial and terminal values of this parameter is calculated by the method described above, and the rise and fall of this parameter throughout the demographic transition are assumed to occur linearly.

respective steady states occurs at the same period. The following figure reports child, adult and old population and the evolution of young age, old age and total dependency ratios for the above described demographic transition scenario.

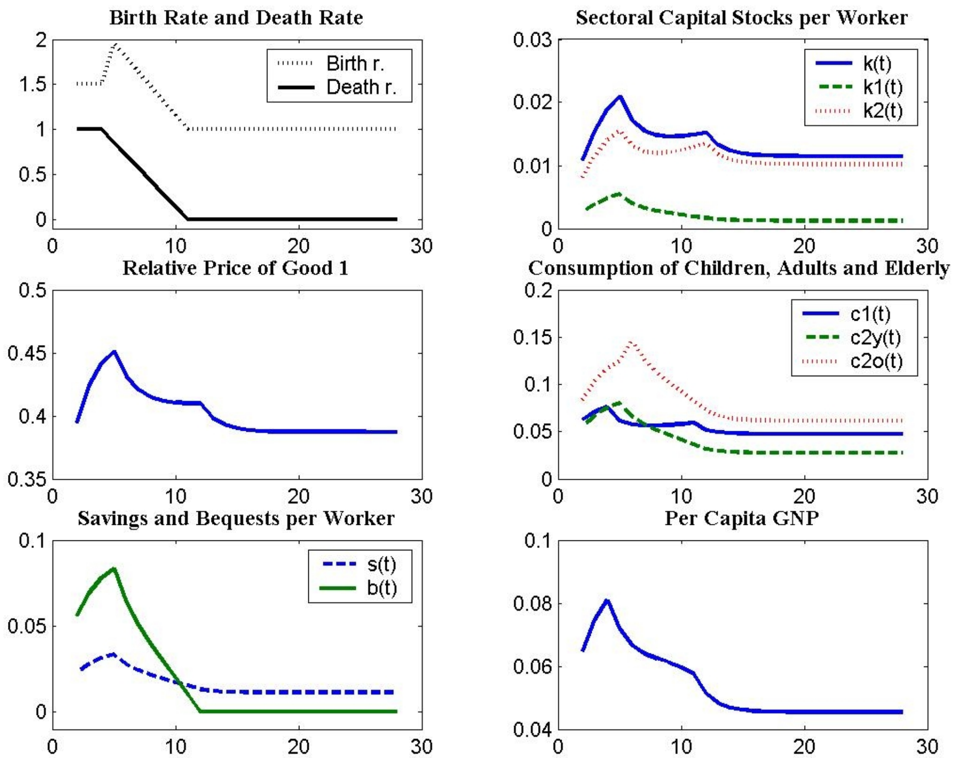


**Figure 4.10: Results of Scenario Sim.2.1.**

As seen in Figure 4.11, after the sudden jump the youth dependency ratio linearly declines while the old age dependency ratio rise. These ratios combined produce a downward swing in the total dependency ratio which is defined as the ratios of child and old population to adult population, i.e., working population.

Before the transition takes place, where $n$ is constant at 1.5 and $1-\lambda$ is constant at one, the economy tends to converge to a higher capital-labor ratio. Immediately after the demographic transition kicks in, the sudden increase in $n$ conjoined with the decline in adult mortality causes a sharp fall in $p$ and $k$. After the decline in $n$ takes start, the rate of fall in $p$ and $k$ gradually becomes more moderate and completely dies out at the time the steady state fertility and adult mortality are reached.

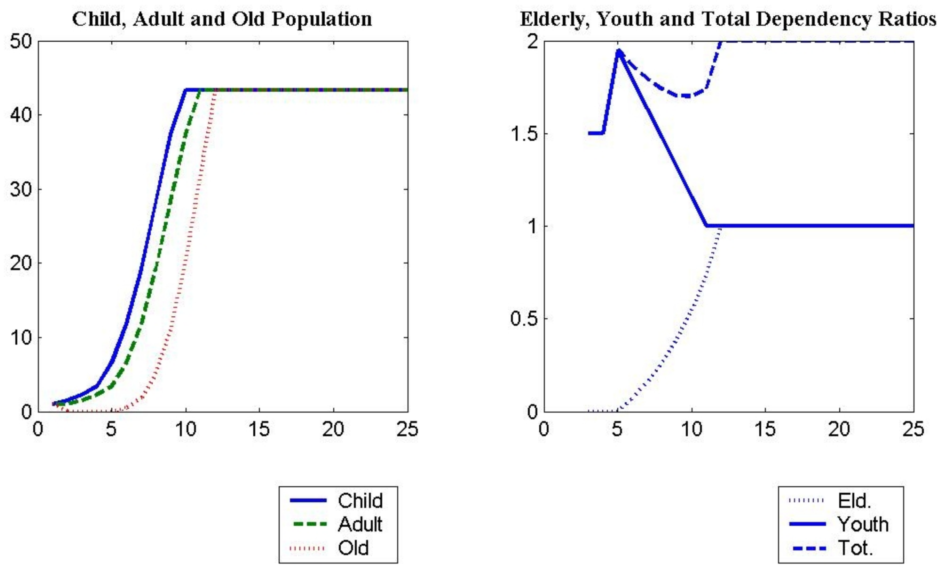


**Figure 4.11: Population Dynamics for Scenario Sim. 2.1**

During this period of falling fertility and adult mortality, two counteracting forces affect the economy in opposite directions. As demonstrated by previous numerical exercises, while the declining birth rates have a welfare enhancing impact, declining death rates work in the other direction. Since in the end of the demographic transition the economy is carried onto a higher plateau of capital-labor ratio compared to the one corresponding to the steady state population dynamics, the convergence to steady state capital per worker starting at around period 12 occurs in the form of a second phase of decline.

Throughout the transition, the negative income effect due to the falling relative price of good 1 pulls down the savings. Falling savings per worker, together with the temporary rise in $n$ and the decline in adult mortality, result in a very sharp decline in bequests per worker. Later on, despite the increasing rental rate and declining fertility, bequests continue to fall as a consequence of falling savings and increasing survival rate among adults. As shown in Figure 4.10, with the convergence of adult mortality to zero, bequests reach and stay constant at zero. In the middle-right panel, the time paths of consumption per child, adult and old are plotted. In line with the overlapping nature of the model, the fall in consumption per child in the fifth period is followed by a fall in consumption per adult in the sixth period, and finally consumption by old falls starting with the seventh period. Looking at these values allows one to trace the negative impact of the income deterioration experienced by a representative agent born at the fifth period until her death. Finally, the time path of GNP per capita summarizes the overall impact of the demographic transition on the economy. Although there is short interval of increasing per capita GNP at the initial stages of the transition, after the transition the total output per capita end up at level that is below the initial level.

## 4.4 Summary

In this chapter, the results obtained from the numeric solution of the basic autarky model under various types of demographic scenarios have been presented and discussed. The main focus of simulation exercises has been the impact of different mortality dynamics on the price ratio and capital stock per worker, because analytical solution under positive mortality was not available due to complexity of the problem.

One of the important conclusions of this chapter is that under a setting where adult mortality rate is not correctly anticipated by agents, a rise in mortality increases the capital-labor ratio and lowers the relative price of the capital intensive good (here good 2) in the economy. Increasing mortality results in a lower elderly dependency ratio. Since old consume only good 2, the share of total demand for good 2 in the total consumption demand (demand for good 1 plus demand for good 2) declines, pulling down the relative the price of good 2, i.e., increasing $p$. In the determination of the response of per worker capital to higher mortality, there are mainly two channels simultaneously at work. One of them is the positive income effect due to rising wages as a consequence of rising $p$. Due to this income effect, savings and first period consumption also increase. This wage income effect is accompanied by the second channel where increasing mortality rates lowers the share of adults who survive to old age. Thus, a higher proportion of adults leave bequests to their offsprings. Therefore, given a constant birth rate, the amount of bequests received by each worker rise both because of higher savings per worker -the first channel- and because of falling share of old survivors. The second important observation is that the gap between child and adult consumptions narrows down when fertility rate increases. Although higher fertility reduces both $c_1$ and $c_{2y}$, the downward shift in child consumption $c_1$ is higher in magnitude.

Finally, the change in adult mortality has important implications for the labor allocation between capital and labor intensive sectors. When adult mortality is zero, the sectoral labor allocation is indifferent to changes in fertility. Other things constant, a rise in adult mortality lowers the effective labor hours employed in the capital intensive sector and raises those employed in the labor intensive sector. On the other hand, when adult mortality is positive, this time a fall in fertility lowers the effective labor employed in the capital intensive sector and raises that employed in the labor intensive sector. This qualitative difference between the effects of fertility on $l_{1s}$ and $l_{2s}$ under zero versus positive adult mortality can be explained by the presence of bequests as an intergenerational wealth transfer mechnanism. Under zero adult mortality bequests are zero and there is no intergenerational wealth transfer. Any rise (fall) in the relative price and capital per worker due to a fall (rise) in fertility affects the sectoral labor allocation in opposite directions. In particular, a rise (fall) in $l_{1s}$ due to an increase (decrease) in $p_s$ is totally offset by a fall (rise) in $l_{1s}$ due to an increase (decrease) in $k_s$, preventing the sectoral labor allocation to change in response to a change in fertility. Under positive adult mortality, however, the intergenerational wealth transfer from deceased old to adults not only magnifies the impact of a rise (fall) in the relative price on future generations by causing a feedback effect from bequests to next period's saving and consumption, but also enhances the substitution effect between two goods. Given a positive rate of adult mortality, higher $p_s$ due to lower $n_s$ increases the wealth of the representative agent through income and bequest effects. Some part of total consumption demand is diverted from good 2 towards good 1, because the income gap between adults and elderly widens in favor of adults, and elderly spend their savings only on good 2, whereas adults spend some part of their wealth on good 1 for their children. Table 4.2 reports the steady state values for some model variables under four different demographic scenarios. The arrows in the table show the direction of change in steady state values with respect to the corresponding benchmark case. A comparison between Sim.1.2 and Base 1.1 confirms that under zero adult mortality sectoral labor allocation is neutral to changes in fertility. Comparing Sim. 1.1 and Base 1.1 verifies that higher mortality alters the labor allocation in favor of the labor intensive sector.

**Table 4.2: Steady State Values under Four Demographic Scenarios**

| | **Base 1.1** | **Sim. 1.1 (Compared with Base 1.1)** | | **Sim.1.2 (Compared with Base 1.1)** | | **Alternative Scenario (Compared with Sim. 1.1)** | |
|---|---|---|---|---|---|---|---|
| **Variable** | $\lambda = 1$ $n = 1.65$ | $\lambda = 0.4$ $n = 1.65$ | | $\lambda = 1$ $n = 1$ | | $\lambda = 0.4$ $n = 1$ | |
| $p_s$ | 0.3170 | 0.3961 | ↑ | 0.3873 | ↑ | 0.4959 | ↑ |
| $k_s$ | 0.0042 | 0.0117 | ↑ | 0.0115 | ↑ | 0.0356 | ↑ |
| $l_{1s}$ | 0.2240 | 0.3569 | ↑ | 0.2240 | ↔ | 0.3746 | ↑ |
| $l_{2s}$ | 0.7760 | 0.6431 | ↓ | 0.7760 | ↔ | 0.6254 | ↓ |
| $w_s$ | 0.0348 | 0.0607 | ↑ | 0.0573 | ↑ | 0.1064 | ↑ |
| $r_s$ | 7.1940 | 4.1213 | ↓ | 4.3600 | ↓ | 2.3500 | ↓ |
| $s_s$ | 0.0070 | 0.0193 | ↑ | 0.0115 | ↑ | 0.0356 | ↑ |
| $b_s$ | 0.0000 | 0.0360 | ↑ | 0.0000 | ↔ | 0.0715 | ↑ |
| $x_{1s}$ | 0.0351 | 0.0781 | ↑ | 0.0474 | ↑ | 0.1148 | ↑ |
| $x_{2s}$ | 0.0539 | 0.0780 | ↑ | 0.0890 | ↑ | 0.1331 | ↑ |
| $c_{1s}$ | 0.0213 | 0.0473 | ↑ | 0.0474 | ↑ | 0.1148 | ↑ |
| $c_{2ys}$ | 0.0167 | 0.0464 | ↑ | 0.0275 | ↑ | 0.0854 | ↑ |
| $c_{2os}$ | 0.0569 | 0.0990 | ↑ | 0.0615 | ↑ | 0.1192 | ↑ |
| $U_s$ | 0.0271 | 0.0638 | ↑ | 0.0385 | ↑ | 0.1003 | ↑ |

Lastly, the ‘Alternative Scenario’ compared with Sim. 1.1 shows that the impact of fertility change on labor allocation is no longer zero when adult mortality and thus bequests per worker are positive.

In the rest of the chapter some other demographic scenarios have been analyzed. These involve gradually falling fertility and mortality rates and a demographic transition where both population parameters starts to fall with a time lag and converge to pre-determined constants at the same period. These simulations intend to see the shape of the transition path to the steady state, when population parameters evolve throughout time. The demographic transition scenario yielding results summarized in Figure 4.10 clearly shows that the fall in birth rates accompanying the previously started fall in death rates enables the economy to recover after a slowdown and carries it to a higher plateau of capital accumulation.

As pointed out before, the major shortcoming of the basic model is the standing assumption that workers are not able to correctly anticipate their likelihood of survival to the next period. Hence they cannot adjust their saving rate in response to changing mortality. The model introduced in the next chapter will fix this shortcoming and investigate the economic consequences of demographic change by comparing them with the ones obtained under the basic model.

# CHAPTER 5:

# The Model with Anticipated Adult Mortality and Simulation Experiments

This chapter presents an alternative model to the basic model to analyze the implications of mortality anticipation. Unlike in the basic model where agents behave as if adult mortality is zero, agents in this model can perfectly anticipate the mortality rates they face and act accordingly. As it will be demonstrated in the following section, the representative agent will adjust her saving rate in response to changes in mortality. Like before, the closed form solutions cannot be attained when mortality rate is positive. Hence, the analytical solution of the long-run equilibrium will be obtained under the restriction of zero mortality. For the case of positive mortality, we will confine ourselves to the numeric solution of the model followed by various demographic simulations and their interpretations. The focus of the chapter will then be on the qualitative differences between results from the basic model and the 'anticipation' model in view of the responses to mortality shocks.

## 5.1 Dynamic equilibrium of the model with anticipation for the case of zero adult mortality

When we relax the assumption that agents do not anticipate their probability of death, $\lambda_t$ enters the utility function as an exogenous variable. Although the consumer gets utility from first period consumption with certainty, the expected second period utility will simply depend on the probability of survival to the old age. With the following representation of the utility function, the problem of the consumer becomes

$$
\begin{aligned}
&\underset{c_{1t}, c_{2yt}, c_{2ot+1}}{Max} \; U_t = \theta \ln\left(n_t c_{1t}\right) + \left(1-\theta\right)\ln\left(c_{2yt}\right) + \lambda_t \frac{1}{1+\rho}\ln\left(c_{2ot+1}\right) \; s.t. \\
&n_t p_t c_{1t} + c_{2yt} + \frac{c_{2ot+1}}{(1+r_{t+1})} = w_t l + \frac{\left(1-\lambda_{t-1}\right)}{n_{t-1}}\left(1+r_t\right)s_t \\
&c_{1t} > 0,\; c_{2yt} > 0,\; c_{2ot+1} > 0
\end{aligned}
\tag{5.1}
$$

where $\frac{1}{1+\rho}$ is the discount factor with $\rho \in \left(0,1\right)$ being the discount rate. This formulation makes the consumer value second period consumption more as the survival rate increases or the discount rate decreases. The saving rate will increase as the agents' prospects to survive into the old age increase. Unlike the utility function given in (3.5), the above representation of utility involves increasing returns to scale since $\lambda_t \frac{1}{1+\rho} > 0$ except in the case of zero survival rate.

One complication that should be addressed in this formulation is the possibility of saving less than a sufficient amount to cover the second period consumption. The question is whether the agents will be discouraged to save a sufficient portion of their wealth in their working age so as to satisfy their optimal consumption demand in the old age, even when they know that they will die with a high probability, i.e., when $\lambda_t$ is very low. In fact, the answer to this question is no, given that the life-time budget constraint in (5.1) holds. This formulation of the budget constraint ensures that consumption plans for the first and second period are consistent with each other. This involves the implicit assumption that agents are not allowed to take the risk of running a budget deficit at the terminal period of their lives, because they do not have a borrowing channel available. This assumption is fairly reasonable since one can argue that the old aged neither can rely on their children to finance their budget deficit, nor can they get loans from financial markets. The former option is not available since there is usually no legal mechanism that can make the descendants of a deceased generation to pay the debt of that generation. The second option is not likely because old agents of any period, who are destined to die at the end of that period with certainty and who do not earn a wage income anymore, would not be considered credit-worthy even if a credit market had existed. Hence, working agents should and do decide on their savings so as to eliminate the possiblity of running a budget deficit in the age of retirement. An alternative justification for retaining the budget constraint above can be made by assuming that a representative agent is risk averse in the sense that he or she is not willing to take the risk of being bankrupt. This kind of attitude towards risk can be supported by arguing that social and individual costs of bankruptcy are so high that gambling over your lifetime uncertainty never pays off. In other words, it usually is too costly for an individual to save less and consume more in adulthood just by considering the possibility that he may not be lucky enough to survive into the old age, since doing so may imply at the end that his savings do not cover your his intended expenses in the old age.

Since the utility function in (5.1) is different from the one given in the basic model with 'no anticipation' of survival probability, the optimality conditions and hence the demand functions of consumers in this OLG economy will be different. Using the first order conditions of the optimization problem in (5.1) and the budget constraint, we can derive per child, per worker and per old consumptions in the economy at time $t$ as

$$c_{1t} = \frac{\theta(1+\rho)}{(1+\rho+\lambda_t)}(w_t l + b_{t-1})\frac{1}{n_t p_t} \tag{5.2}$$

$$c_{2yt} = \frac{(1-\theta)(1+\rho)}{1+\rho+\lambda_t}(w_t l + b_{t-1}) \tag{5.3}$$

and

$$c_{2ot+1} = \frac{\lambda_t}{1+\rho+\lambda_t}(w_t l + b_{t-1})(1+r_{t+1}) \tag{5.4}$$

respectively. Since per worker saving is equal to $s_t = w_t l + b_{t-1} - n_t p_t c_{1t} - c_{2yt}$, it can be shown by using (5.2) and (5.3) that the saving rate in the economy at time $t$ is equal to $\frac{\lambda_t}{1+\rho+\lambda_t}$ which is increasing in survival rate $\lambda_t$ and decreasing in the discount rate $\rho$.

There will not be any change in the problem of the firms producing good 1 and good 2, and therefore the first order conditions derived from the profit maximization problems of both sectors will be identical with the one in the basic model with no mortality anticipation. Hence, as it has been done in Chapter 3, the closed form equilibrium solutions of this model under zero adult mortality can be found by combining the optimality conditions of consumers' and producers' problems via market clearence condition for good 1: $x_{1t} = n_t c_{1t}$. Since it is assumed that $\lambda_t = 1$, the consumption demand for good 1 becomes

$$c_{1t} = \frac{Am_1(1-\alpha)l\delta^{\alpha}}{n_t} p_t^{\frac{\alpha}{\beta-\alpha}} \tag{5.5}$$

when the expression for the wage rate is plugged in (5.2). In equation (5.5), $m_{1t} = \frac{\theta(1+\rho)}{1+\rho+\lambda_t} = \frac{\theta(1+\rho)}{2+\rho} = m_1$ denotes the marginal propensity to consume for good 1 out of the wage income. $m_1$ is constant since the adult mortality is assumed to be constant at unity. We know from Chapter 3 that per worker output of good 1 is equal to

$$x_{1t} = A\left[\frac{\varepsilon l}{\varepsilon-\delta} - \frac{1}{\varepsilon-\delta} k_t p_t^{\frac{1}{\alpha-\beta}}\right]\delta^{\alpha} p_t^{\frac{\alpha}{\beta-\alpha}} \tag{5.6}$$

Then, the market clearance condition implies

$$\begin{aligned} &\frac{1}{\varepsilon-\delta} k_t p_t^{\frac{1}{\alpha-\beta}} = \frac{\varepsilon l}{\varepsilon-\delta} - m_1(1-\alpha)l \Rightarrow \\ &k_t = \left[\varepsilon - (1-\alpha)(\varepsilon-\delta)m_1\right] l p_t^{\frac{1}{\beta-\alpha}} \end{aligned} \tag{5.7}$$

Savings at time $t$ will be equal to new investment on capital stock at time $t+1$. This condition can be expressed as

$$k_{t+1} = \frac{s_t}{n_t} = \frac{A(1-\alpha)(mps)l\delta^{\alpha} p_t^{\frac{\beta}{\beta-\alpha}}}{n_t} \tag{5.8}$$

where $mps_t = \frac{\lambda_t}{1+\rho+\lambda_t} = \frac{1}{2+\rho} = mps$ is equal to the saving rate or the marginal propensity to save out of wage income. Combining (5.7) and (5.8) the law of motion for the price ratio can be expressed as

$$\frac{p_{t+1}}{p_t^{\beta}} = \left\{\frac{A(1-\alpha)(mps)\delta^{\alpha}}{n_t\phi_1}\right\}^{\beta-\alpha} \tag{5.9}$$

where $\phi_1 = \varepsilon\left(1-\left(1-\alpha\right)m_1\right)+\delta\left(1-\alpha\right)m_1$. It is easy to calculate the steady state price ratio by simply setting $p_t = p_{t+1} = p_s$ and having

$$p_s = \left(\frac{\phi_2}{n_s\phi_1}\right)^{\frac{\beta-\alpha}{1-\beta}} \tag{5.10}$$

where $\phi_2 = A\left(1-\alpha\right)(mps)\delta^{\alpha}$. Alternatively, everything can be expressed in terms of per worker capital stock as

$$\frac{k_{t+1}}{k_t^{\beta}} = \left(\frac{\phi_2}{n_t\phi_1^{\beta}}\right)l^{1-\beta} \tag{5.11}$$

and at the steady state, per worker capital stock will be

$$k_s = \left(\frac{\phi_2}{n_s\phi_1}\right)^{\frac{1}{1-\beta}}\phi_1 l \tag{5.12}$$

Using $p_s$ and $k_s$ the long-run closed form solutions of other variables can be calculated as follows. The real wage rate at the steady state is decreasing in the level of fertility $n_s$ and increasing in the rate of saving $mps$.

$$w_s = A\left(1-\alpha\right)\delta^{\alpha}\left(\frac{\phi_2}{n_s\phi_1}\right)^{\frac{\beta}{1-\beta}} \tag{5.13}$$

Unlike the real wage rate, the rental rate is declining in the rate of saving and increasing in fertility rate.

$$r_s = \frac{\alpha}{\left(1-\alpha\right)(mps)\delta}n_s\phi_1 \tag{5.14}$$

The sectoral per worker capital stocks at the steady state are given as follows.

$$k_{1s} = \frac{\delta l}{\varepsilon-\delta}\left(\frac{\phi_2}{n_s\phi_1}\right)^{\frac{1}{1-\beta}}\left(\varepsilon-\phi_1\right) \tag{5.15}$$

$$k_{2s} = \frac{\varepsilon l}{\varepsilon - \delta}\left(\frac{\phi_2}{n_s \phi_1}\right)^{\frac{1}{1-\beta}}(\phi_1 - \delta) \tag{5.16}$$

The magnitudes of the long-run per worker capital stocks employed in both sectors is decreasing in the rate of long-run fertility and increasing in the saving rate $mps$.

To see why first note that $\delta = \left(\frac{\alpha}{\beta}\right)^{\frac{\beta}{\beta-\alpha}}\left(\frac{1-\beta}{1-\alpha}\right)^{\frac{\beta-1}{\beta-\alpha}}$ and $\varepsilon = \left(\frac{\alpha}{\beta}\right)^{\frac{\alpha}{\beta-\alpha}}\left(\frac{1-\beta}{1-\alpha}\right)^{\frac{\alpha-1}{\beta-\alpha}}$ imply that $\delta = \varepsilon\left(\frac{\alpha}{\beta}\right)\left(\frac{1-\beta}{1-\alpha}\right)$. Thus, $\varepsilon - \delta = \varepsilon\left(\frac{\beta-\alpha}{\beta(1-\alpha)}\right) > 0$ since $\beta > \alpha$ by assumption.

Now consider $k_{1s}$, the capital stock employed in sector 1. Using equation (5.15),

$$\frac{\partial k_{1s}}{\partial n_s} = \frac{\delta(\phi_1 - \varepsilon) l}{(\varepsilon - \delta)(1-\beta) n_s}\left(\frac{\phi_2}{n_s \phi_1}\right)^{\frac{1}{1-\beta}}$$

which is negative since $\phi_1 - \varepsilon = (\delta - \varepsilon)(1-\alpha) m_1 < 0$ as $\varepsilon > \delta$ and $m_1 > 0$. On the other hand,

$$\frac{\partial k_{1s}}{\partial mps} = \frac{A(1-\alpha)\delta^{\alpha+1}(\varepsilon - \phi_1) l}{(\varepsilon - \delta)(1-\beta) n_s \phi_1}\left(\frac{\phi_2}{n_s \phi_1}\right)^{\frac{\beta}{1-\beta}} > 0$$

since $\varepsilon - \phi_1 > 0$. Next consider $k_{2s}$, the steady state per worker capital stock employed in sector 2. Using equation (5.16),

$$\frac{\partial k_{2s}}{\partial n_s} = \frac{\varepsilon(\delta - \phi_1) l}{(\varepsilon - \delta)(1-\beta) n_s}\left(\frac{\phi_2}{n_s \phi_1}\right)^{\frac{1}{1-\beta}}$$

which is again negative since $\delta - \phi_1 = (\delta - \varepsilon)\alpha m_1 < 0$.Finally,

$$\frac{\partial k_{2s}}{\partial mps} = \frac{\varepsilon(\phi_1 - \delta) l}{(1-\beta) n_s \phi_1}\left(\frac{\phi_2}{n_s \phi_1}\right)^{\frac{\beta}{1-\beta}} > 0$$

since $\phi_1 - \delta > 0$.

The sectoral labor allocation at the steady state is unaffected by the steady state fertility rate. Given constant technology and preference parameters, the share of labor employed in each sector converges to levels that are independent of the fertility rate and saving rate.

$$l_{1s} = \frac{l}{\varepsilon - \delta}(\varepsilon - \phi_1) \tag{5.17}$$

$$l_{2s} = \frac{l}{\varepsilon - \delta}(\phi_1 - \delta) \tag{5.18}$$

Steady state per worker output of each sector is given as follows:

$$x_{1s} = \frac{l}{\varepsilon - \delta} A\delta^{\alpha} (\varepsilon - \phi_1) \left( \frac{\phi_2}{n_s \phi_1} \right)^{\frac{\alpha}{1-\beta}} \tag{5.19}$$

$$x_{2s} = \frac{l}{\varepsilon - \delta} A\varepsilon^{\beta} (\phi_1 - \delta) \left( \frac{\phi_2}{n_s \phi_1} \right)^{\frac{\beta}{1-\beta}} \tag{5.20}$$

The output levels are decreasing in fertility rate and increasing in the saving rate by the same logic used in the proof of Corollary 1. Equation (5.21) gives the long-run value of the savings per worker which is negatively related to the number of children per adult and clearly positively related to the saving rate.

$$s_s = \phi_2 l \left( \frac{\phi_2}{n_s \phi_1} \right)^{\frac{\beta}{1-\beta}} \tag{5.21}$$

The consumption per child is increasing in the marginal propensity to consume for good 1, $m_1$, increasing in the saving rate and decreasing in rate of fertility since

$$c_{1s} = \frac{A(1-\alpha)\delta^{\alpha}}{n_s} m_1 l \left( \frac{\phi_2}{n_s \phi_1} \right)^{\frac{\alpha}{1-\beta}} \tag{5.22}$$

The steady state consumption by each adult is given by

$$c_{2ys} = A(1-\alpha)\delta^{\alpha} m_{2y} l \left(\frac{\phi_2}{n_s \phi_1}\right)^{\frac{\beta}{1-\beta}} \tag{5.23}$$

where $m_{2yt} = \frac{(1-\theta)(1+\rho)}{1+\rho+\lambda_t} = \frac{(1-\theta)(1+\rho)}{2+\rho} = m_{2y}$. Like $c_{1s}$, this magnitude is positively related to the marginal propensity to consume for good 2 when adult, $m_{2y}$, and decreasing in fertility rate. Finally, equation (5.24) gives the consumption of good 2 by old.

$$c_{2os} = \left(\frac{\phi_2}{n_s \phi_1}\right)^{\frac{\beta}{1-\beta}} l\left[\phi_2 + A\alpha\delta^{\alpha-1} n_s \phi_1\right] \tag{5.24}$$

The effect of fertility on old consumption is ambiguous like in the model introduced in Chapter 3. More specifically,

$$\frac{\partial c_{2os}}{\partial n_s} \begin{cases} \geq 0 \text{ if } & n_s \geq \left[\dfrac{\delta\beta(1-\alpha)(mps)}{(1-\beta)\alpha\phi_1}\right]^{\frac{1}{2}} \\ < 0 \text{ if } & otherwise \end{cases}$$

## 5.2 Simulation Results under 'Anticipated Mortality' Model

The model described in the previous section is numerically solved for the parameter values given on Table 5.1. Capital share parameters $\alpha$ and $\beta$ in the production functions of sector 1 and sector 2 are equal to the ones in the basic model. Like in the previous model, $\theta$ denotes the share of child consumption in the first period utility of the representative agent. $\rho$ stands for the discount rate of the agent for the second period utility.

**Table 5.1: Production and Preference Parameters for Baseline and Simulation Experiments**

| $\alpha$ | $\beta$ | $\theta$ | $\rho$ | $l$ | $A$ | $b_1$ | $C_1, N_1, O_1$ |
|---|---|---|---|---|---|---|---|
| 0.3 | 0.5 | 0.4 | 0.02 | 1 | 10 | 0 | 1,1,1 |

As it is evident from equation (5.1), this time preference parameter is independent and distinct from the probability of survival. As a general practice, the effective labor per worker is normalized to one. Unlike in the basic model without anticipation effect, in this model, utility function is logarithmic. Therefore, consumption levels below unity result in negative utility values. This poses no real problem, since utility is a purely ordinal measure here. Nonetheless, in order to avoid such negative values, the shift parameter $A$, that governs the total factor productivity, is taken as 10 instead of unity. This creates a scaling-up effect without changing the results in any qualitative sense. Finally, like before, initial level of bequests is assumed to be zero and initial child, worker and old populations are all equal to one. The numeric solution and simulation methodology is identical to the one used in the solution of basic model. The following four subsections presents the outcomes of various demographic experiments summarized in Table 5.2 and discusses their implications.

**Table 5.2: A Snapshot of Simulation Experiments in Chapter 5**

<table>
<tr><td></td><td>Higher Fertility</td><td colspan="5">Higher Mortality</td></tr>
<tr><td></td><td></td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr>
<tr><td>$n$</td><td>1→ 1.2</td><td>1</td><td>1</td><td>1</td><td>3</td><td>1.65</td></tr>
<tr><td>$\lambda$</td><td>1</td><td>1→0.80</td><td>0.40→0.20</td><td>1→0.40</td><td>1→0.40</td><td>1→0.40</td></tr>
<tr><td></td><td>Gradually Declining Mortality</td><td>Demographic Transition</td><td colspan="4">Cyclical Shocks to Mortality</td></tr>
<tr><td>$n$</td><td>1</td><td>2→1</td><td colspan="4">3</td></tr>
<tr><td>$\lambda$</td><td>0.20→0.40</td><td>0.7→0.90</td><td colspan="4">0.30→0.20→0.40→0.1→0.5→0.3</td></tr>
</table>

### 5.2.1 The Effect of Increasing Fertility

Figure 5.1 shows the time paths per worker capital stock, real per capita GNP, effective labor hours and relative price of good 1 under base scenario ($n = 1$ and $\lambda = 1$) and under the simulation scenario ($n = 1.2$ and $\lambda = 1$).

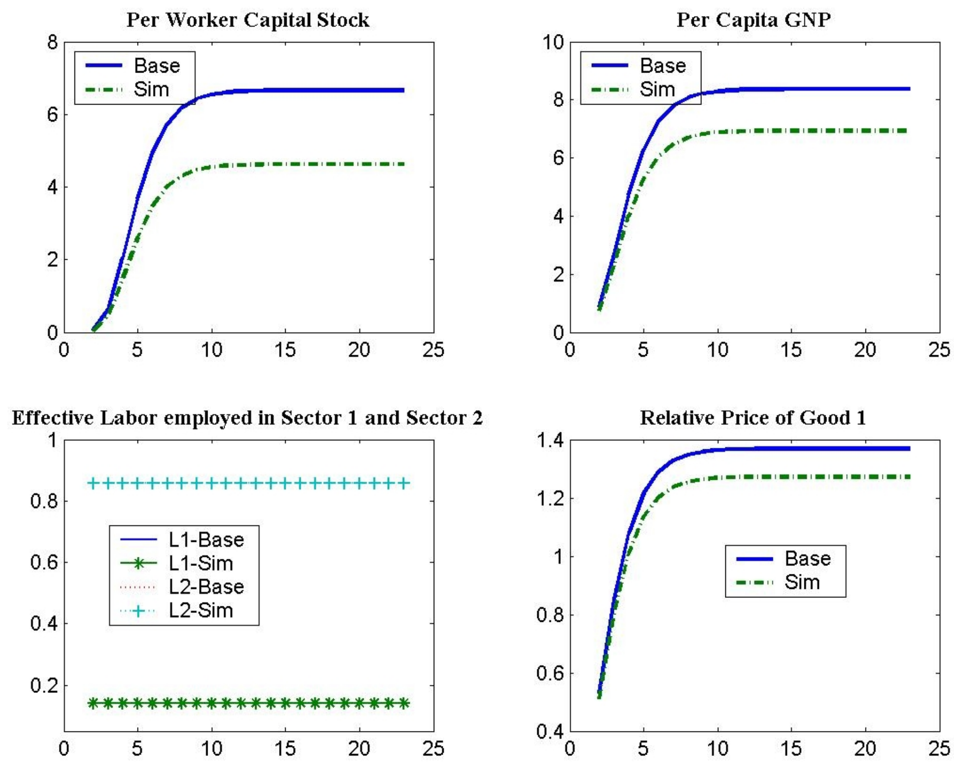


**Figure 5.1: An Increase in Fertility Rate from 1 to 1.2 ($\lambda$ = 1)**

As the analytical solution suggests, the capital stock and the price ratio is decreasing in $n$. The time paths of these variables under the simulation scenario lie below the ones under the base scenario.

As suggested by (5.17) and (5.18), effective labor hours employed in both sectors remain unchanged. In Figure 5.2 , consumption and output levels under low and high fertility rates are compared.

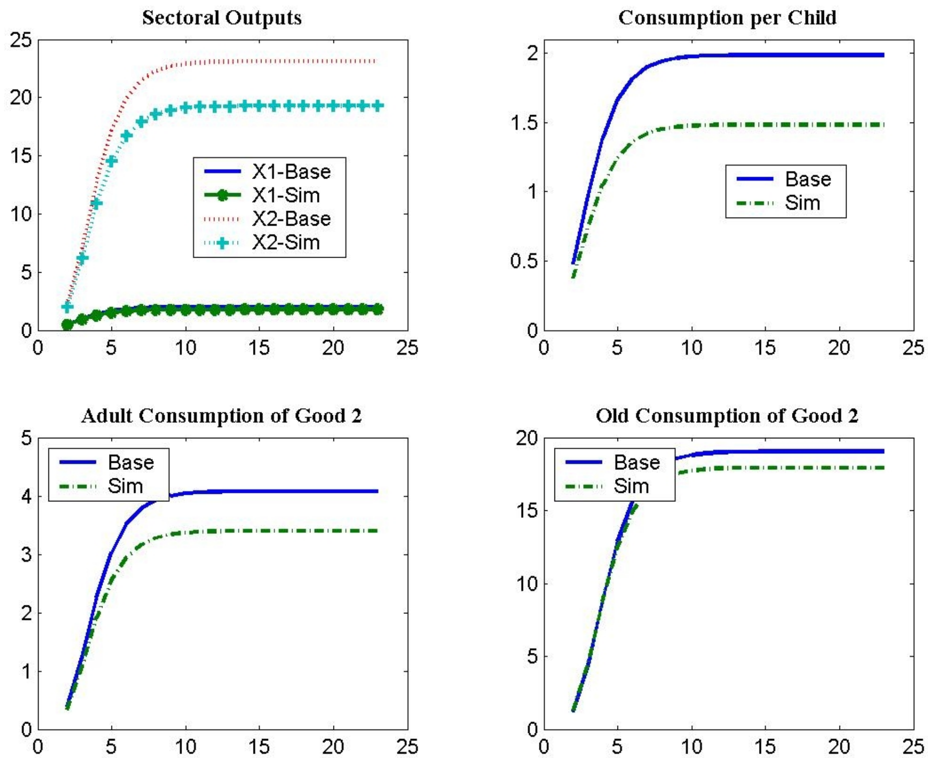


**Figure 5.2: Consumption and Output due to an Increase in Fertility Rate from 1 to 1.2 ( $\lambda$ = 1)**

The consumption of good 1 by each child, the consumption of good 2 by each worker and by each old person all lie below the levels that correspond to the base scenario. Per worker output of good 2 is significantly lower when fertility is higher. Similarly, per worker output of good 1 lies slightly below the baseline level, although this slight change is not seen clearly in the figure. In summary, the effect of a change in fertility on the long-run values of the endogenous variables is qualitatively similar to that of the basic model. However, the impacts of a change in mortality rates can be different under the basic model and under the model with mortality anticipation.

### 5.2.2 The Effect of Increasing Mortality

Changes in the probability of death will affect the second period utility. As mentioned before, this expected utility formulation implies that the mortality rates are perfectly anticipated by the representative agent. The fact that the introduction of this anticipation effect may change the qualitative results attained by the basic model can be seen in the following two figures. Figure 5.3 describes the effects of an increase in adult mortality rate from 0% to 20%, when the fertility rate is constant at one.

In this particular simulation experiment, "Sim" denotes the transition path of the variables under 20% mortality rate, while "Base" stands for the case of zero mortality. The figure suggests that, like under the basic model, higher mortality is welfare improving since the capital accumulation occurs faster. This fact is reflected in per capita real GNP figure. Compared to the baseline scenario, the labor share of sector 1 production increases and that of sector 2 decreases as a consequence.

When the initial death rate is higher, Figure 5.4 reveals that the direction of the impact of an identical decline in mortality is reversed. When the mortality rate jumps from 60% to 80%, while the fertility rate is held constant at one, a surprisingly significant reduction can be observed in the levels of per worker capital stock and per capita GNP.

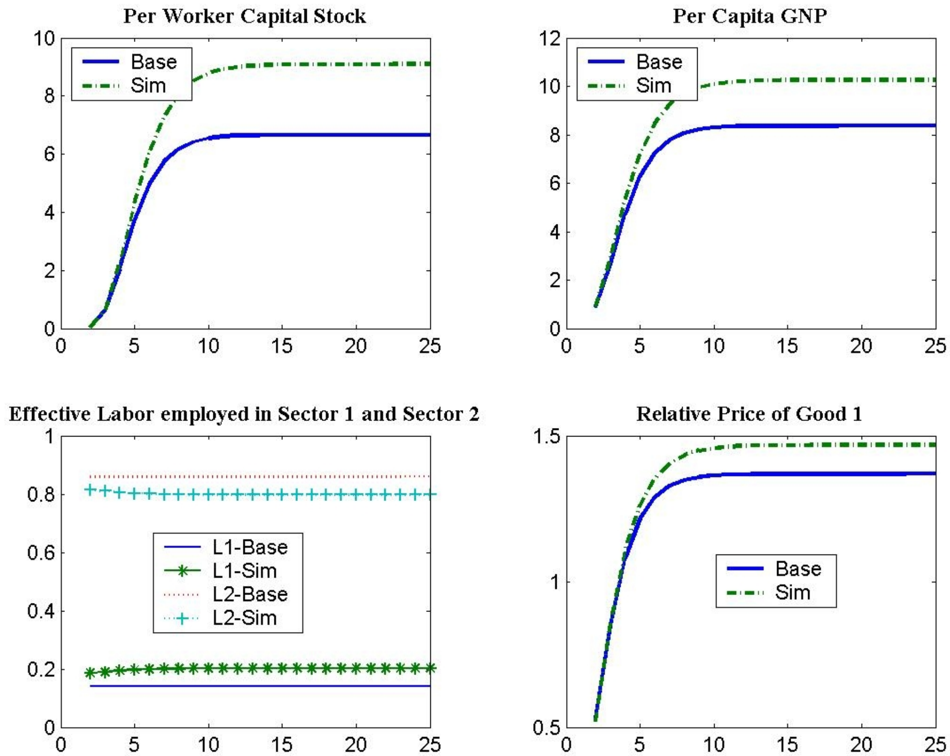


**Figure 5.3: An Increase in Mortality Rate from 0% to 20% (n = 1)**

This finding is in contrast with the findings of the basic model, which does not include the mortality anticipation effect. Hence, the first major difference between this model and the previous one is that in the latter changes in mortality have a unidirectional impact on the economy, while in the former the direction of the impact changes its direction after a certain threshold level of mortality. In other words, the model with anticipation effect is such that at any given birth rate there is a mortality threshold, below which higher mortality is welfare improving and above which it is welfare deteriorating.

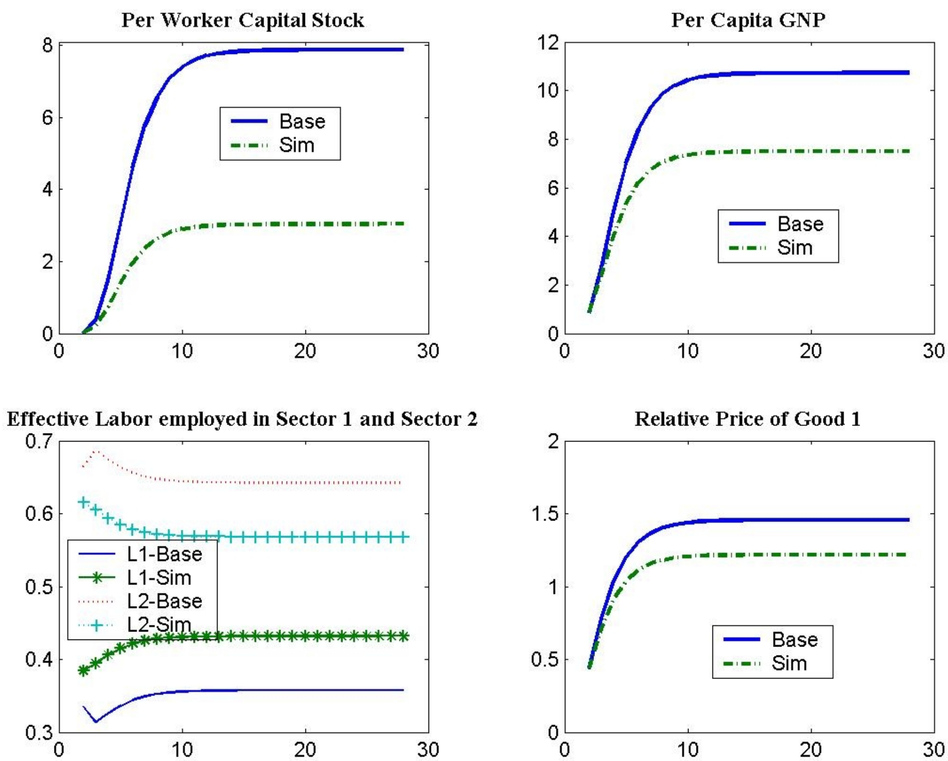


**Figure 5.4: An Increase in Mortality Rate from 60% to 80% (n = 1)**

Figure 5.5 shows the transition of per worker capital stock and the price ratio under two simulations in both of which the mortality rate jumps from 0% to 60%. However, these are distinct simulations since the simulation given in the upper panel involves a fertility rate of one, whereas the one in the lower panel involves a fertility rate of 3.

Under the experiment with the birth rate being equal to one, per worker capital and price ratio stays slightly below the baseline level in the early periods of the transition, but then both of them overtake the corresponding baseline levels at around seventh period and stays above the latter in the rest of the model horizon. In contrast, under the second experiment with $n = 3$, although increasing mortality raises the steady state price ratio very slightly, this price effect is insufficient to offset the negative saving rate effect and to raise the per worker capital above the baseline level associated with no mortality.

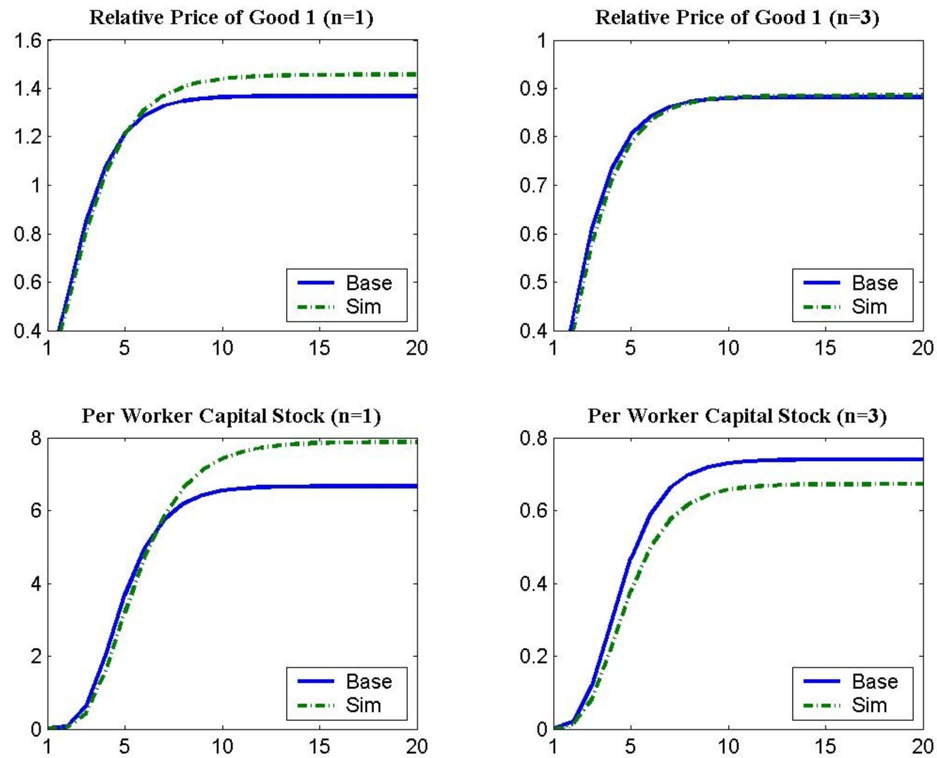


**Figure 5.5: The Impact of an Increase in Mortality from 0% to 60% When Fertility is High vs. Low**

This finding suggests that higher fertility lowers the mortality threshold described above, because when fertility is increased, a rise in mortality becomes less favorable. This implies that relative to low fertility levels, at higher levels of fertility, a smaller initial mortality level is sufficient for the positive impact of an equal rise in mortality to be reversed and become negative. If we consider two countries with identical initial mortality rates and different birth rates, the country experiencing higher birth rates has more incentive to decrease the mortality rate compared to the country experiencing lower birth rates. This outcome is plausible, because a high fertility country has already a big disadvantage in capital accumulation and it needs higher saving rates to stimulate the capital formation. More precisely, trading higher mortality, and thus a higher proportion of adults leaving bequests, for lower mortality and thus higher saving rate and higher per worker savings (due to anticipation effect) is necessary for countries that are on the higher tail of fertility distribution and hence on the lower tail of income distribution. On the other hand, low fertility countries have higher levels of per worker income and savings. Capital accumulation is faster and steady state capital stock is above that of high fertility countries. Therefore the scale effect resulting from a positive shock to mortality rate can much easily overcome the negative impact of a fall in saving rates by increasing the ratio of the deceased adult population to the number of potential bequest receivers of the next generation.

In summary, two major results regarding the autarky model with accurately anticipated rates of mortality and their implications are as follows:

**1-** Unlike in the basic model, the effect of increasing (or decreasing) mortality rate is not unidirectional. The direction of the impact may be positive or negative depending on the particular combination of fertility and mortality rates that govern the population.

**2-** In particular, as the mortality rate among adults rises, the positive impact of a marginal increase in mortality diminishes, and above a threshold level of mortality, this effect becomes negative as verified by the results presented in Figure 5.3 and Figure 5.4.

**3-** As the level of fertility increases, marginal benefit of increasing mortality rate from its initial level becomes less and less pronounced. After a particular fertility level, the initial positive effect turns negative as verified by the comparison in Figure 5.5.

**4-** These results imply that unlike in the basic model, it is not optimal for an economy to have 100% adult mortality. Especially, young societies with higher birth rates should put a higher effort on increasing their expected lifetime so that increasing saving rates helps the economy to take off from its low initial capital stock. As for older societies with low fertility rates, the converse may be true. For them, a higher death rate implies a relatively higher level of bequest per worker and thus higher savings.

Figure 5.6 and Figure 5.7 summarize the effects of an increase in mortality from 0% to 60% on per worker outputs of each sector and consumption of each generation when $n=1$ and $n=3$ respectively. In both figures, it is seen that a shift from low to high mortality regime causes per worker output of good 1 to increase and that of good 2 to decline, since there is an increase in the relative price of good 1. In line with the analytical results, the steady state output and consumption levels are higher under low fertility ($n=1$). The only qualitative difference between the figures is observed in the time paths of old age consumption. In the country with low fertility ($n=1$), per worker capital accumulation and growth of per worker income is faster than in high fertility country with $n=3$. Thus, starting from the same initial capital stock, at each period in time, per worker savings and capital in low-fertility country is above that in high-fertility country. The impact of positive shocks of the same magnitude to mortality rate in each country at time $t$ can be traced by analyzing the following equations:

$$s_t = \frac{\lambda_t}{1+\rho+\lambda_t}\left(w_t l + b_{t-1}\right) \tag{5.25}$$

$$b_{t-1} = \frac{1-\lambda_{t-1}}{n_{t-1}} s_{t-1}\left(1+r_t\right) \tag{5.26}$$

where $s_t$ denotes the savings per worker and $b_{t-1}$ denotes the amount of bequests received by each worker living at time $t$.

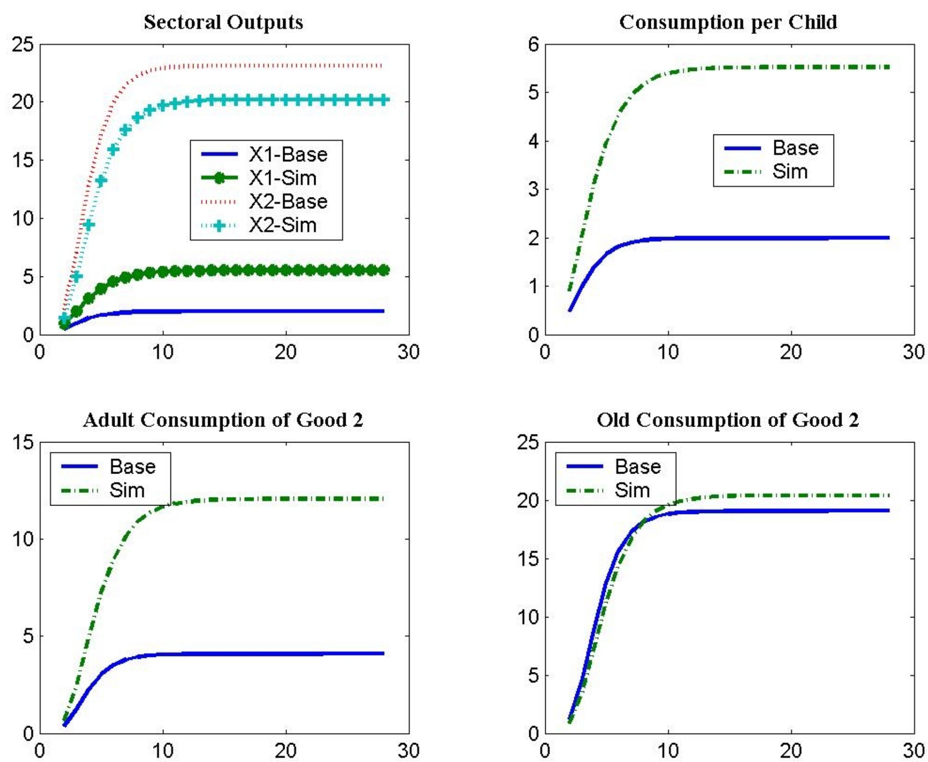


**Figure 5.6: Sectoral Outputs per worker and Consumption when Mortality increases from 0% to 60% (n = 1)**

Differentiating (5.25) with respect to $\lambda_t$ the following equation is attained:

$$\frac{\partial s_t}{\partial \lambda_t} = \left(w_t l + b_{t-1}\right)\frac{1+\rho}{\left(1+\rho+\lambda_t\right)^2} + \frac{\lambda_t}{1+\rho+\lambda_t}\left[\frac{\partial p_t}{\partial \lambda_t}\frac{\partial w_t}{\partial p_t} l + \frac{\partial p_t}{\partial \lambda_t}\frac{\partial r_t}{\partial p_t}\frac{\partial b_{t-1}}{\partial r_t}\right] \tag{5.27}$$

where $\frac{\partial b_{t-1}}{\partial r_t} = \frac{1-\lambda_{t-1}}{n_{t-1}} s_{t-1}$. The first term on the RHS –showing the saving rate effect- is clearly positive. It is evident from Figure 5.5 that throughout the first seven periods, this positive shock to mortality rate slightly lowers $p$, i.e. $\frac{\partial p_t}{\partial \lambda_t} > 0$. Since $\frac{\partial w_t}{\partial p_t} > 0$, the first term in brackets –showing the wage effect- is positive. On the other hand, $\frac{\partial r_t}{\partial p_t} < 0$ and $\frac{\partial b_{t-1}}{\partial r_t} > 0$ imply that the second term in brackets –showing the bequest effect- is negative.

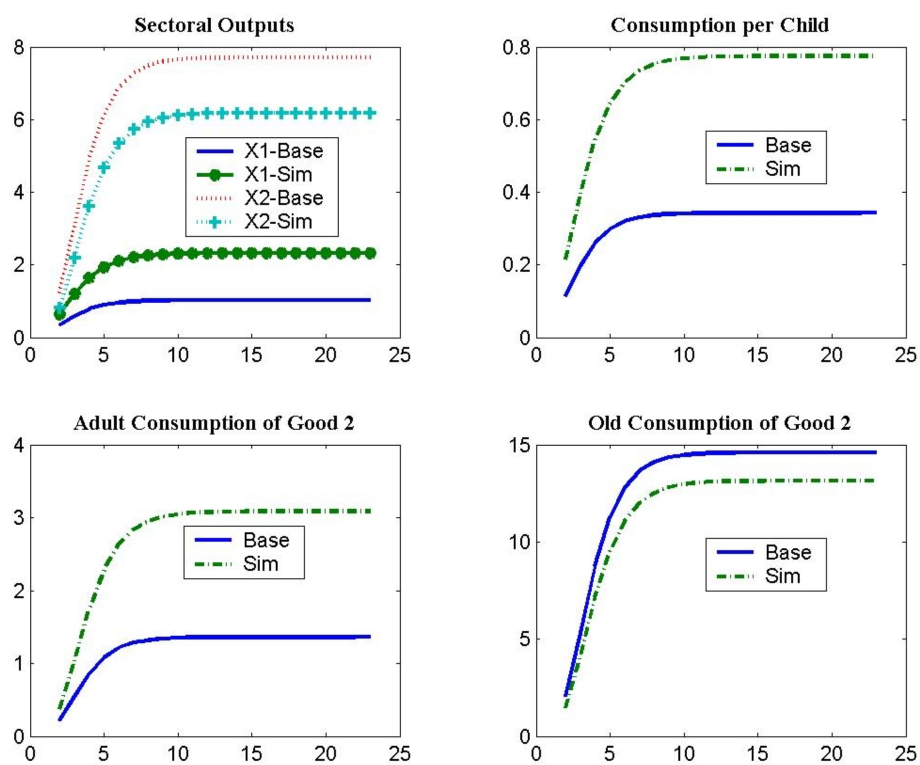


**Figure 5.7: Sectoral Outputs per worker and Consumption when Mortality increases from 0% to 60% (n = 3).**

Hence, the net impact of a decline in the survival rate on savings may vary depending on the relative magnitudes of the two terms in the bracket and the magnitude of the saving rate effect. During the first few periods, like in the high-fertility country ($n = 3$), the positive income effect in the low-fertility country dominates the negative bequest effect, which results in a slightly lower level of capital stock compared to the base scenario with $\lambda = 1$ (see Figure 5.5). This implies that the positive wage effect and positive saving rate effect together dominate the negative bequest effect. However, since the savings keep on increasing towards the steady state, after some time, the net impact will change direction because the positive impact of rental rates on bequests, given by $\frac{\partial b_{t-1}}{\partial r_t} = \frac{1-\lambda_{t-1}}{n_{t-1}} s_{t-1}$, becomes larger and larger, the greater $s_{t-1}$ becomes and the lower the birth rate $n_{t-1} = n$ is. Consequently, in a low-fertility country, the negative bequest effect starts to dominate the positive wage and saving rate effects, meaning that $\frac{\partial s_t}{\partial \lambda_t} < 0$, i.e., savings are increasing in the rate of mortality. Thus, as shown in the lower right panel of Figure 5.6, after a certain period, higher mortality makes the saving per worker –hence the consumption by old- exceed its level in the base scenario. On the other hand, Figure 5.5 and Figure 5.7 above suggest that the fertility rate in the high-fertility country is so high that the time paths of the relative price, capital stock and old consumption shift all the way down to a level below the corresponding time paths under the base scenario. The sectoral per worker outputs under the basic model and under the 'anticipation model' respond to changes in mortality differently. Figure 5.8 compares the changes in the transition paths of per worker sectoral outputs $x_1$ and $x_2$ in response to a rise in steady state mortality from

0% to 60% ($n$ = 1.65) both for the basic model and for the 'mortality anticipation' model.[31]

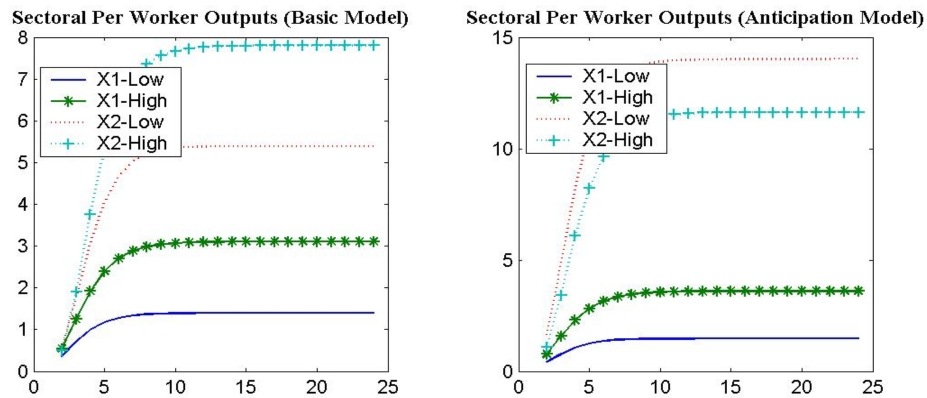


**Figure 5.8: Responses of Sectoral Outputs to Increasing Mortality under Basic vs. Anticipation Models (n = 1.65)**

Since the two models are different in terms of the formulation of consumer preferences through different utility functions, it is normal to observe that the steady state values of sectoral outputs vary across the two models. The interesting point is that the response of sector 2 output is qualitatively different under each model. Under the basic model, presented in Chapter 3, both $x_1$ and $x_2$ increase due to increasing mortality rate, while under the 'mortality anticipation' model, presented in this chapter, $x_1$ increases, but $x_2$ decreases in response to an increase in steady state mortality. The reason for this qualitative difference between the responses of the output of capital-intensive good lies in the difference between the formulations of the saving rate in the models. In the basic model, saving rate is constant at $1-\mu$ while in the 'anticipation' model saving rate $\frac{\lambda_t}{1+\rho+\lambda_t}$ is dynamic and decreasing in the rate of mortality due to the mortality anticipation component of the model. This

[31] Both models are solved by setting the production shift parameter A equal to 10.

component allows workers to reduce their saving rates in the face of an expected fall in their survival probability, hence also reducing per worker savings and bequests. This reduction in savings leads to a decline in the demand for good 2 by old and corresponding reduction in the investment good demanded by sector 2, which consequently causes a fall in the output of good 2.

### 5.2.3 The Effects of Gradually Declining Mortality and Demographic Transition

Figure 5.9 summarizes the results of the anticipation counterpart of Simulation 1.3 which has been conducted under the basic model in Chapter 4. According to the demographic scenario shortly denoted by ‘Sim’, death rate starts to fall linearly after the fourth period from 0.80 to 0.60. In the scenario denoted by ‘Base’, death rate stays constant at its initial level of 0.80. In both scenarios birth rate is assumed to be at the replacement level.

At around period 5, the time path of per worker capital, per capita GNP, sectoral effective labors and the relative price under ‘Sim’ starts to diverge from the path under ‘Base’. As mortality declines further, the gap between per worker capital stocks, relative prices and per capita GNPs continue growing until the steady states are reached. Decreasing mortality leads to an increase in the saving rate, which makes the representative agent substitute some part of $c_{1t}$ and $c_{2yt}$ for $c_{2ot+1}$. At the same time, a higher relative price for good 1 raises the wage income, creating a positive income effect and increasing both savings and current consumption.

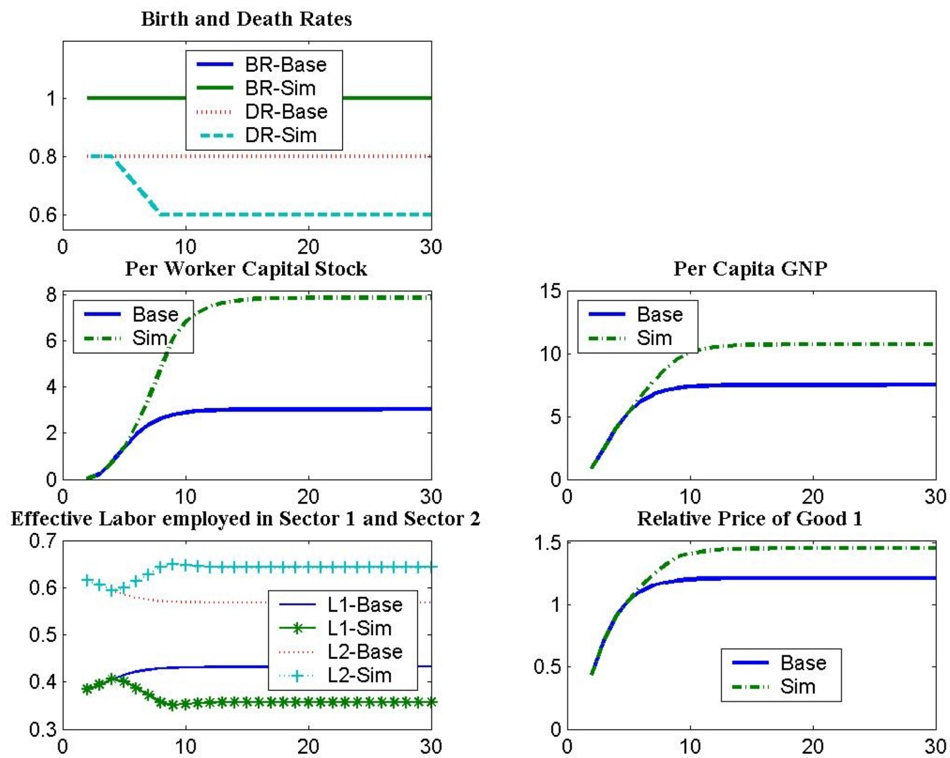


**Figure 5.9: The Effect of Gradually Declining Mortality (n = 1)**

The inter-temporal substitution and income effects together increase the savings and capital stock relative to the 'Base'. In response to mortality decline, some part of the unit effective labor is diverted from sector 1 to be employed in sector 2, so that sector 2 can increase its production capacity to meet increasing investment and consumption demands for good 2. Figure 5.9 demonstrates the possibility that lower mortality may be welfare improving for the society, if adult mortality rate can be accurately anticipated by agents. These results are in contrast with the results of Sim.1.3 presented in Chapter 4, where declining mortality slowed down capital accumulation and resulted in a lower relative price and capital stock compared to 'Base' scenario.

Figure 5.10 compares the time paths of the economy with and without the demographic transition. The uppermost panels in the figure show the evolution of the population under a demographic transition (Sim) and no demographic transition (Base).

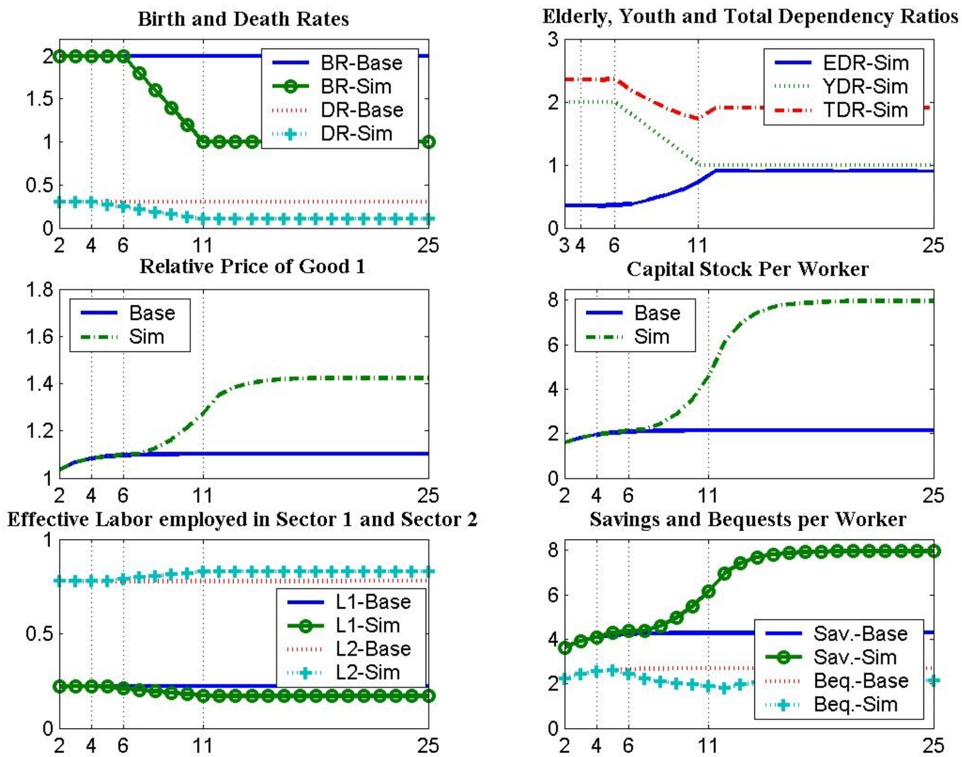


**Figure 5.10: Demographic Transition under 'Anticipation'**

In the fourth period, mortality rate starts to fall from 0.3 to 0.1, and two periods later fertility rate starts its sharp decline from two children to one child per adult. Convergence of both population parameters occurs at the same period, namely in period 11. This demographic scenario is interesting because it exploits two important characteristics of the 'anticipation model': **(i)** From the experiments in Figure 5.3 and Figure 5.4, we know that an increase (decrease) in mortality rate of a certain magnitude may lower (raise) per worker capital if the initial mortality rate is sufficiently high, and may raise (lower) per worker capital if the initial mortality rate is sufficiently low, **(ii)** Figure 5.5 has demonstrated that the same type of shock to mortality rate may produce qualitatively different results depending on the fertility rate governing the population. By **(ii)** one can argue that, a decline in fertility in the face of declining mortality will tend to slow down or even reverse the direction of the impact of falling mortality rate. In particular, at the early stages of demographic transition, since fertility rate is still high, a decline in mortality will be more valuable and contribute more to the capital formation compared to later stages of demographic transition where fertility is already low. If, in the later periods of the demographic transition, fertility rate drops below a certain level, the decline in mortality may even lower the capital per worker. Since the fall in mortality is linear, the change in mortality is of the same magnitude in each period. Then, keeping other things constant, one can argue by **(i)** that during the initial periods of the transition when mortality rate is still high, a possibly positive impact of a decline in mortality rate will be higher than that of an identical decline occurring at later periods. The direction of this impact may turn negative if the mortality drops below a certain threshold under this demographic transition. These two dynamics of the demographic transition are simultaneously at work and they imply that the higher initial fertility and mortality rates are, the faster is the transition of the economy towards a higher steady state capital per worker.

Looking at Figure 5.10, it can be noticed that after period 7, the relative price suddenly takes off which pulls up savings per worker by increasing wages. Although savings increase starting from period 6-, bequests per worker $b$ decline until around period 12, and then it converges to the steady state with a small increase (see the lower right panel in Figure 5.10). The reasons behind falling bequests per worker until period 12 are (1) the shrinking proportion of deceased workers i.e., increasing old age dependency, who leave bequests as a consequence of falling mortality rate and (2) the decline in rental rates as a consequence of decreasing relative price of capital-intensive good (good 2). While increasing saving per worker due to price and saving rate effects and decreasing return on savings due to negative price effect are influencing bequests in opposite directions, the same is true for the negative scale effect emanating from increasing survival rate and for the positive scale effect due to declining fertility. The response of consumption by old to the demographic transition is different from the responses of child and adult consumption, because old consumption is not only governed by life-time wealth, i.e., $wl+b$, of the agent but also by the course of rental rates.

### 5.2.4 The Effects of Cyclical Shocks to Mortality

Since the basic model introduced in Chapter 3 and the 'anticipation model' introduced in this chapter differ in terms of the impact of mortality rate to economic variables, it is worth comparing the behaviors of each model under cyclical shocks to mortality rate. For this purpose, each model is numerically solved[32] under the population scenario described below.

[32] For a better comparison, both models are solved by setting the production shift parameter A equal to 10.

Figure 5.11 describes the cyclical shocks to mortality as two consecutive cycles in the form of boom-bust-boom-bust. Having remained constant at 70% until the fourth period, mortality rate jumps to 80% and then falls down to 60%. Then, it increases to 90% reaching its peak point and decreases down to 50% before returning to its initial level of 70%.

Figure 5.12 compares the responses of the basic and anticipation models to this cyclical shock. The first thing to note is the difference between the amplitudes of the responses to the shocks. Clearly, the 'anticipation' model is more sensitive and more responsive to the shocks, whereas in the basic model the fluctuations around the balanced growth path are much smoother.

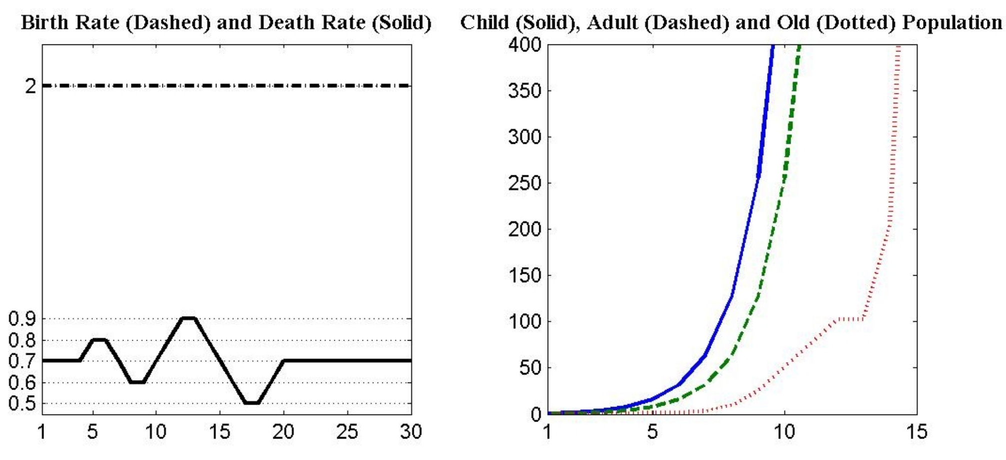


**Figure 5.11: Cyclical Shocks to Mortality Rate and Population by Generations**

The results suggest that immediately after the first positive shock at period 5 raises mortality from 70% to 80%, the relative price under the 'anticipation' model $p_{ant}$ falls causing a similar decline in per worker capital stock $k_{ant}$. Since per capita real

GNP is highly correlated with $k_{ant}$ and $p_{ant}$, it also declines. Under the basic model, however, instead of a decline in $k_{basic}$ and $p_{basic}$, a further rise in both variables can be noticed. Until the end of the cyclical shocks, the capital stock and relative price in each model move in opposite directions.

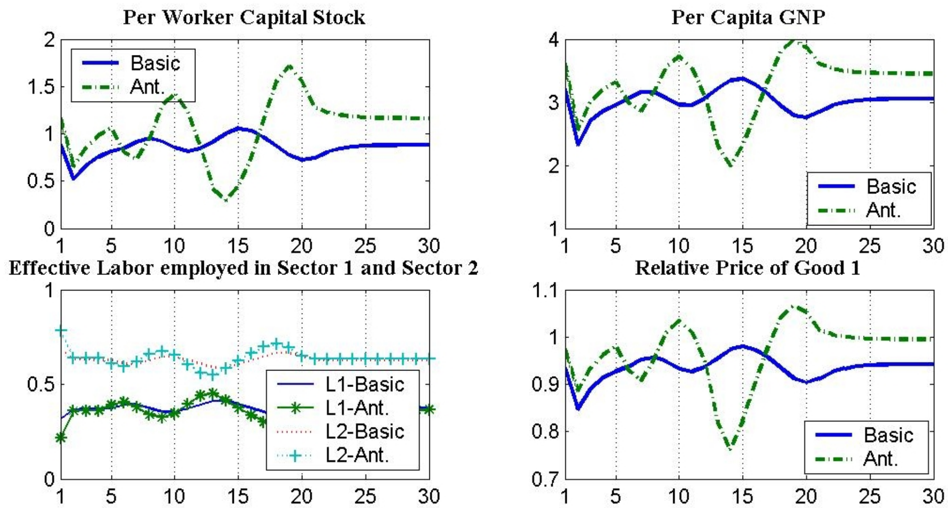


**Figure 5.12: The Effects of Cyclical Shocks to Mortality Rate under the 'Basic Model' vs. 'Anticipation Model' (n = 3)**

In the basic model, capital accumulation accelerates during the upturns in mortality and decelerates during downturns, while the opposite is true under the 'anticipation' model. Therefore, it is of critical importance to know whether these shocks are anticipated or not while analyzing the business cycles caused by demographic shocks. Figure 5.13 summarizes the comparison of the responses in sectoral outputs and consumption by each generation. After the first positive shock to mortality, the outputs of good 1 and good 2 under the 'anticipation' model declines and with the following negative shock they recover. The opposite happens under the basic model. The time paths of child and adult consumptions are almost identical in shape. The old consumption is significantly more responsive under the 'anticipation' model than under the basic model.

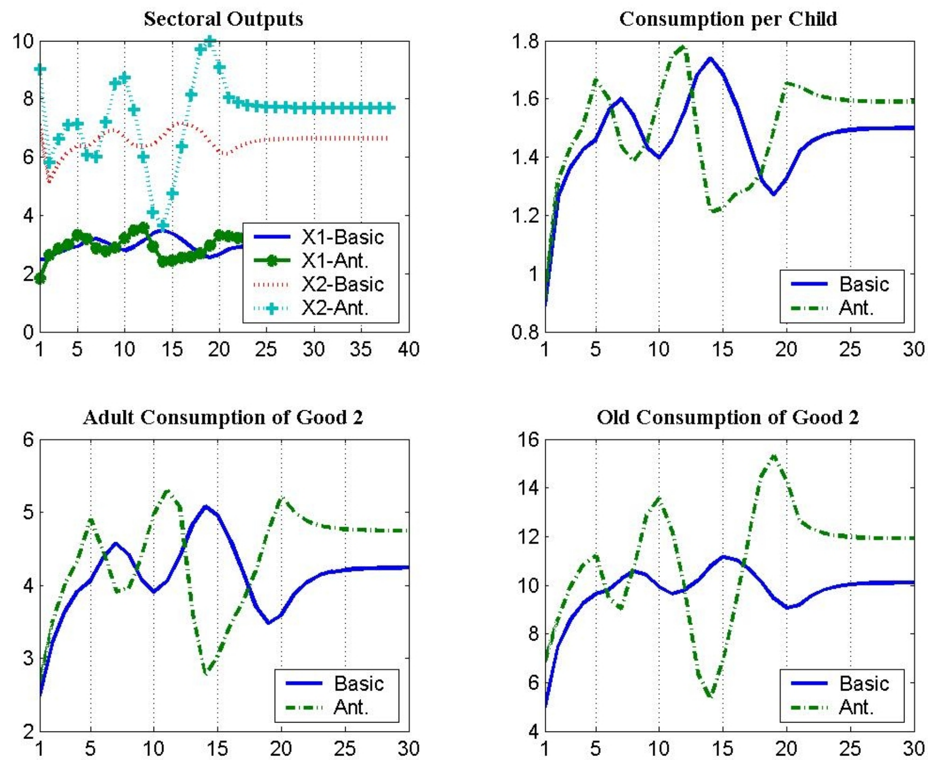


**Figure 5.13: The Effects of Cyclical Shocks to Mortality Rate under the 'Basic Model' vs. 'Anticipation Model' (n = 3) (Cont'd)**

Consequently, the pattern implied by sectoral outputs and per worker capital is also true for consumption levels: $c_1, c_{2y}$ and $c_{2o}$ are decreasing in the rate of mortality when agents can perfectly anticipate the adult mortality and increasing in the rate of mortality when they cannot anticipate it and behave as if it is equal to zero.

## 5.3 Summary

It has been argued in this chapter that relaxing the assumption of unanticipated mortality changes the consumption behavior of agents and alters the responses of capital and relative price to demographic change. In particular, the saving rate becomes a function of the survival rate and the time preference parameter, instead of being a constant like in the basic model. The analytical solutions for the long-run equilibrium under zero adult mortality have shown that a shift in fertility produces qualitatively identical results in each model. This has also been verified by simulation exercises.

More interestingly, it has been demonstrated that a certain change in mortality rate do not always affect the capital stock and other variables in the same direction, if mortality can be anticipated. The direction of the effect may be different depending on the rate of fertility at the time of the shock and on the initial level of mortality to which the shock is applied. More specifically, a fall in mortality rate may increase per worker capital and the relative price of good 1 if fertility rate is sufficiently high or alternatively, if the initial rate of mortality is already sufficiently high. This is the main characteristic of the 'anticipation' model which distinguishes it from the basic model, where welfare is increasing in mortality rate regardless of the level of fertility and mortality.

The rest of the chapter has dealt with the short to medium-run effects of various demographic scenarios such as linearly declining mortality rates, a demographic transition scenario and cyclical shocks to mortality. Main findings can be summarized as follows:

**(i)** The ‘anticipation’ model is more responsive to mortality shocks due to the inclusion of the changing saving rate effect as an extra component.

**(ii)** Each model responds to a certain mortality shock in opposite directions. While in the basic model higher mortality is welfare enhancing, that is not necessarily the case in the ‘anticipation’ model. In a more realistic fashion, the positive marginal impact of increasing mortality diminishes as the level of mortality increases and turns negative after a certain threshold level of mortality. The interaction of mortality with fertility rate is also a determining factor. Once again the marginal benefit of an increase in mortality is decreasing in the level of fertility and after a certain threshold the increase in mortality may be welfare reducing.

After a close investigation of the effects of fertility and mortality dynamics under autarky using two different types of models, the next chapter will introduce free trade and contrast the two models under free trade.

# CHAPTER 6:

# Trade between Two Countries: The Basic and Anticipation Models under Free Trade

## 6.1 The Basic Model under Free Trade

Two hypothetical countries $N$ and $S$ are identical in every respect except their population dynamics. $n_t^N$, $n_t^S$ are the number of children born by each adult living in $N$ and $S$ respectively, while $\lambda_t^N$ and $\lambda_t^S$ denote the probabilities of survival of adults at time $t$ in $N$ and $S$. These countries constitute our hypothetical world, and they trade with each other under a dynamic Heckscher-Ohlin framework. Under free trade, the goods market clearance conditions in the case of autarky will be replaced by global goods market clearance conditions. Markets for each good will clear at the relative price that equate total supply of each good to the total demand. The modified conditions for good 1 and good 2 are

$$X_{1t}^N + X_{1t}^S = C_{1t}^N + C_{1t}^S \tag{6.1}$$

and

$$\left(X_{2t}^{N}+X_{2t}^{S}\right)+\left(K_{t}^{N}+K_{t}^{S}\right)=\left(C_{2yt}^{N}+C_{2yt}^{S}\right)+\left(C_{2ot}^{N}+C_{2ot}^{S}\right)+\left(K_{t+1}^{N}+K_{t+1}^{S}\right) \tag{6.2}$$

Equation (6.1) can be expressed in per worker terms as

$$N_{t}^{N}\left(x_{1t}^{N}-n_{t}^{N}c_{1t}^{N}\right)+N_{t}^{S}\left(x_{1t}^{S}-n_{t}^{S}c_{1t}^{S}\right)=0 \tag{6.3}$$

which is equivalent to

$$N_{1}^{N}\left(\prod_{i=1}^{t-1}n_{i}^{N}\right)\left(x_{1t}^{N}-n_{t}^{N}c_{1t}^{N}\right)+N_{1}^{S}\left(\prod_{i=1}^{t-1}n_{i}^{S}\right)\left(x_{1t}^{S}-n_{t}^{S}c_{1t}^{S}\right)=0 \tag{6.4}$$

Equation (6.2) can be expressed in per worker terms as

$$\begin{aligned}&\left[N_{t}^{N}\left(x_{2t}^{N}+k_{t}^{N}-c_{2yt}^{N}\right)-N_{t-1}^{N}\lambda_{t-1}^{N}c_{2ot}^{N}-N_{t+1}^{N}k_{t+1}^{N}\right]+\\&\left[N_{t}^{S}\left(x_{2t}^{S}+k_{t}^{S}-c_{2yt}^{S}\right)-N_{t-1}^{S}\lambda_{t-1}^{S}c_{2ot}^{S}-N_{t+1}^{S}k_{t+1}^{S}\right]=0\end{aligned} \tag{6.5}$$

which is equivalent to

$$\begin{aligned}&N_{1}^{N}\left(\prod_{i=1}^{t-2}n_{i}^{N}\right)\left[n_{t-1}^{N}\left(x_{2t}^{N}+k_{t}^{N}-c_{2yt}^{N}\right)-\lambda_{t-1}^{N}c_{2ot}^{N}-n_{t-1}^{N}n_{t}^{N}k_{t+1}^{N}\right]+\\&N_{1}^{S}\left(\prod_{i=1}^{t-2}n_{i}^{S}\right)\left[n_{t-1}^{S}\left(x_{2t}^{S}+k_{t}^{S}-c_{2yt}^{S}\right)-\lambda_{t-1}^{S}c_{2ot}^{S}-n_{t-1}^{S}n_{t}^{S}k_{t+1}^{S}\right]=0\end{aligned} \tag{6.6}$$

where $N_1^N$ and $N_1^S$ denote the initial adult populations in $N$ and $S$.

Assuming that free trade will lead to an equalization of the relative prices across both countries, we impose

$$\tilde{p}_{t}^{N}=\tilde{p}_{t}^{S}=\tilde{p}_{t},\ \forall t\geq 1 \tag{6.7}$$

In chapter 3, it has been shown that the rental and wage rates are functions of the price ratio and constant production parameters. Since it is assumed that the trading countries possess identical production technologies, from (6.7) it follows that

$$\tilde{r}_{t}^{N}=\tilde{r}_{t}^{S}=\tilde{r}_{t}\quad\text{and}\quad\tilde{w}_{t}^{N}=\tilde{w}_{t}^{S}=\tilde{w}_{t},\ \forall t\geq 1 \tag{6.8}$$

Unlike in Jelassi (2004) and Sayan (2005), the amounts of child, adult and old consumptions are not necessarily equal with each other across the trading countries[33] This difference is due to the presence of mortality rate in this model. If the mortality rates and/or fertility rates across countries are different from each other, the level of bequests and therefore wealth levels will be different across countries, which in turn will be reflected in consumption levels. More precisely, by the open expression for bequests per worker given in equation (3.44), unless $\frac{1-\lambda_t^N}{n_t^N} = \frac{1-\lambda_t^S}{n_t^S}$ at each period, consumption levels may be different across $N$ and $S$. The second factor is the unequal rates of fertility across countries. Even though the ratio of mortality rate to birth rate may be equal across the countries, a difference in fertility rates at a point in time, may lead to a difference in child consumption across $N$ and $S$ at that point in time. This fact can be seen from the demand function of child consumption given in (3.10). The fact that differences between consumption levels across the trading parties may arise without any difference in consumer preferences improves upon the models in Sayan (2005) and Jelassi (2004) by making this model more realistic in this sense.

The laws of motion for the capital stocks in each country are identical to the case of autarky, where saving at time $t$ is equal to investment at time $t+1$ in each country so that we have

$$k_{t+1}^N = \frac{(1-\mu)\left(\tilde{w}_t l + \tilde{b}_{t-1}^N\right)}{n_t^N} \tag{6.9}$$

and

[33] The OLG models in Sayan (2005) and Jelassi (2004) involve two generations and two goods, where each of the goods is consumed by both generations. In this model, however, good 1 is consumed only by children, while good 2 is consumed only by adults and old.

$$k_{t+1}^{S}=\frac{(1-\mu)\left(\tilde{w}_t l+\tilde{b}_{t-1}^{S}\right)}{n_t^{S}} \tag{6.10}$$

As was the case in the autarky model, the trade model with the above-described modifications does not lend itself to closed form solutions for the long-run equilibrium. However, in order to provide a partial insight for the long-run dynamics of the model under free trade, an analytic solution will be obtained by assuming away the adult mortality. The next section will present the derivation of dynamic equilibrium under free trade, when adults survive to old age with certainty.

### 6.1.1 The Dynamic Equilibrium under Full Certainty of Survival into Old Age

The market clearance condition for good 1 given in (6.3) is not affected from the assumption of 'no mortality', while the market clearance condition for good 2 should be rewritten by setting $\lambda_t^N=\lambda_t^S=1$. In other words, survival probabilities are assumed to be equal to one in each country. Then (6.5) can be rewritten as

$$\begin{aligned}&\left[N_t^N\left(x_{2t}^N+k_t^N-c_{2yt}^N\right)-N_{t-1}^N c_{2ot}^N-N_{t+1}^N k_{t+1}^N\right]+\\&\left[N_t^S\left(x_{2t}^S+k_t^S-c_{2yt}^S\right)-N_{t-1}^S c_{2ot}^S-N_{t+1}^S k_{t+1}^S\right]=0\end{aligned} \tag{6.11}$$

By Walras' Law, we can confine ourselves to the market clearance for good 1, since it automatically implies market clearance for good 2. Since there will not be any change in the optimality conditions coming from profit and utility maximization problems under free trade in both countries, the expressions showing optimal consumption, labor and capital demands will be identical with the case of autarky. Then, to derive the steady state price ratio, first we substitute $x_{1t}$ given in (3.35) and $c_{1t}$ given in (3.10) into (6.3) for each country. To distinguish this model from the autarky model, the variables under free trade will be denoted by upper script tilda in the rest of the chapter. Thus, the resulting equation after the necessary substitutions is

$$A\delta^{\alpha}\tilde{p}_t^{\frac{\beta}{\beta-\alpha}}\left(N_t^S\tilde{l}_{1t}^S + N_t^N\tilde{l}_{1t}^N\right) = \mu\theta\tilde{w}_t l\left(N_t^S + N_t^N\right) \tag{6.12}$$

Using (3.31), (3.32) and (3.34) we can plug in $\tilde{l}_{1t}^S$, $\tilde{l}_{1t}^N$ and $\tilde{w}_t$ in the above equation to get the following equation

$$\frac{1}{\varepsilon-\delta}\left[\varepsilon l\left(N_t^N + N_t^S\right) - \tilde{p}_t^{\frac{1}{\alpha-\beta}}\left(N_t^N\tilde{k}_t^N + N_t^S\tilde{k}_t^S\right)\right] = \mu\theta(1-\alpha)l\left(N_t^N + N_t^S\right) \tag{6.13}$$

Letting $\tilde{N}_t = N_t^N + N_t^S$ be the total adult population in both countries, letting $\tilde{K}_t^S$ and $\tilde{K}_t^N$ denote the total capital stocks in country $S$ and country $N$ and rearranging the terms in (6.13) we have

$$\left(\tilde{K}_t^N + \tilde{K}_t^S\right)\tilde{p}_t^{\frac{1}{\alpha-\beta}} = \Phi_1 l\left(N_t^N + N_t^S\right) \tag{6.14}$$

where $\Phi_1 = \varepsilon\left(1-\mu\theta(1-\alpha)\right) + \delta\mu\theta(1-\alpha)$ according to the notation introduced in Chapter 3. Since bequests are ruled out by the assumption that adults survive into old age with certainty, capital market clearance conditions for each country are

$$\tilde{k}_{t+1}^N = \frac{(1-\mu)\tilde{w}_t l}{n_t^N} \tag{6.15}$$

and

$$\tilde{k}_{t+1}^S = \frac{(1-\mu)\tilde{w}_t l}{n_t^S} \tag{6.16}$$

Using these conditions, the following relationship between per worker capital stocks (capital-labor ratios) in $N$ and $S$ is attained

$$\tilde{k}_{t+1}^{S} = \left(\frac{n_t^N}{n_t^S}\right)\tilde{k}_{t+1}^{N} \tag{6.17}$$

Equation (6.17) tells that the capital-labor ratio of the country, which had a lower fertility rate in the previous period than the other country, will be higher than the capital-labor ratio of the other country. Now writing (6.15) and (6.16) for time t, expressing total capital stock as

$$\tilde{K}_t^S + \tilde{K}_t^N = (1-\mu)\tilde{w}_{t-1}l\left(N_{t-1}^S + N_{t-1}^N\right) = A(1-\alpha)(1-\mu)\delta^{\alpha}l\tilde{N}_{t-1}\tilde{p}_{t-1}^{\frac{\beta}{\beta-\alpha}} \tag{6.18}$$

and finally using the expression for wages in (3.34) once again, the law of motion for the relative price of good 1 is written as

$$\left(\frac{\tilde{p}_t}{\tilde{p}_{t-1}^{\beta}}\right)^{\frac{1}{\alpha-\beta}} = (1+g_t)\left(\frac{\Phi_1}{\Phi_2}\right) \tag{6.19}$$

where $\Phi_2 = A(1-\mu)(1-\alpha)\delta^{\alpha}$, and $g_t = \dfrac{\tilde{N}_t}{\tilde{N}_{t-1}} - 1$ is the growth rate of the total adult population in both countries. Alternatively, we can derive the difference equation giving the optimal rule for the dynamics of capital stock per worker. First using (6.17) total capital stock in the trading countries can be expressed in terms of per worker capital stock of only one country as follows:

$$\tilde{K}_t^N + \tilde{K}_t^S = \left[N_t^S\left(\frac{n_{t-1}^N}{n_{t-1}^S}\right) + N_t^N\right]\tilde{k}_t^N = \left(N_{t-1}^S n_{t-1}^N + N_t^N\right)\tilde{k}_t^N$$

$$\Rightarrow \tilde{K}_t^N + \tilde{K}_t^S = \tilde{N}_{t-1}n_{t-1}^i\tilde{k}_t^i \quad \text{for each } i \in \{N,S\} \tag{6.20}$$

Using (6.18) the world price ratio can be written in terms of the total capital stock

$$\tilde{p}_{t-1} = \left[ \frac{\tilde{K}_t^S + \tilde{K}_t^N}{\tilde{N}_{t-1}\Phi_2 l} \right]^{\frac{\beta-\alpha}{\beta}} \tag{6.21}$$

This expression tells that when $\beta > \alpha$, world relative price is increasing in the level of world savings and decreasing in total worker population of the world. Writing $p_t$ from (6.21), equation (6.14) can be rewritten as

$$\left[ \frac{\left(\tilde{K}_t^S + \tilde{K}_t^N\right)^\beta}{\tilde{K}_{t+1}^S + \tilde{K}_{t+1}^N} \right] = \frac{\Phi_1{}^\beta}{\Phi_2}\left(\tilde{N}_t l\right)^{\beta-1} \tag{6.22}$$

Finally substituting (6.20) into (6.22) for $i = N,S$ we can obtain the difference equation for per worker capital in $N$ and $S$ as

$$\left[ \frac{\left(\tilde{N}_{t-1} n_{t-1}^i \tilde{k}_t^i\right)^\beta}{\tilde{N}_t n_t^i \tilde{k}_{t+1}^i} \right] = \frac{\Phi_1{}^\beta}{\Phi_2}\left(\tilde{N}_t l\right)^{\beta-1} \tag{6.23}$$

for each $i \in \{N,S\}$.

By definition, at the steady state $\tilde{k}_{t+1}^i = \tilde{k}_t^i = \tilde{k}_s$ and $\tilde{p}_t = \tilde{p}_{t-1} = \tilde{p}_s$ should hold. Then, using (6.19) the steady state price ratio can be written as

$$\tilde{p}_s = \left[ \left(1+g_s\right)\left(\frac{\Phi_1}{\Phi_2}\right) \right]^{\frac{\alpha-\beta}{1-\beta}} \tag{6.24}$$

where $g_s$ is the growth rate of adult population (labor force) at the steady state, i.e., $1+g_s = \lim_{t\to\infty}\left(1+g_t\right)$. Note that $\frac{\Phi_1}{\Phi_2} > 0$ and $\beta > \alpha$ imply that the steady state world relative price of labor intensive consumption good is decreasing in the rate of growth of global labor force. This result is in line with the autarky result that $p_s$ is decreasing in fertility rate. Similarly using (6.23) and imposing $\tilde{k}_{t+1}^i = \tilde{k}_t^i = \tilde{k}_s$ and $n_{t+1}^i = n_t^i = n_s^i$ as preconditions for steady state, the level of per worker capital stock in each country at the steady state will be equal to

$$\tilde{k}_s^i = \frac{\left(1+g_s\right)^{\frac{\beta}{\beta-1}}}{n_s^i}\left(\frac{\Phi_1^{\beta}}{\Phi_2}\right)^{\frac{1}{\beta-1}} l \tag{6.25}$$

for $i \in \{N,S\}$.

Using the definition of the growth rate of global labor force we can write

$$1+g_t = \frac{\tilde{N}_s}{\tilde{N}_{s-1}} = \frac{N_1^S\left(\prod_{i=1}^{s-1} n_i^s\right) + N_1^N\left(\prod_{i=1}^{s-1} n_i^N\right)}{N_1^S\left(\prod_{i=1}^{s-2} n_i^s\right) + N_1^N\left(\prod_{i=1}^{s-2} n_i^N\right)} \tag{6.26}$$

where $N_1^i$ is the initial worker population in country $i \in \{N,S\}$. For the steady state of the trade model to exist, there should be a period $t^* \in [1,\infty)$ such that $n_t^N = n^N$ and $n_t^S = n^S \;\; \forall t \geq t^*$ for some constant fertility rates $n^N$ and $n^S$.Then assuming that $n^S > n^N$ and using (6.26) we have

$$1+g_s = \lim_{t\to\infty}\left(1+g_t\right) = \lim_{t\to\infty}\frac{N_{t^*}^S\left(n^S\right)^{t-t^*-1} + N_{t^*}^N\left(n^N\right)^{t-t^*-1}}{N_{t^*}^S\left(n^S\right)^{t-t^*-2} + N_{t^*}^N\left(n^N\right)^{t-t^*-2}} =$$

$$= \lim_{t\to\infty}\left\{\frac{1+\frac{N_{t^*}^N}{N_{t^*}^S}\underbrace{\left(\frac{n^N}{n^S}\right)^{t-t^*-1}}_{\searrow 0}}{\frac{1}{n^S}+\frac{N_{t^*}^N}{N_{t^*}^S}\frac{1}{n^S}\underbrace{\left(\frac{n^N}{n^S}\right)^{t-t^*-2}}_{\searrow 0}}\right\} = n^S \tag{6.27}$$

Equation (6.27) tells that in the long-run and under constant rates of fertility, country $S$, with its higher fertility rate, will act as a large country whose population dynamics will determine the world relative price. This can be verified by plugging $\left(1+g_s\right)$ into equation (6.24) so that we are left with

$$\tilde{p}_s = \left[ n^S \left( \frac{\Phi_1}{\Phi_2} \right) \right]^{\frac{\beta-\alpha}{\beta-1}} \tag{6.28}$$

which is identical to the steady state level of relative price given in (3.61) for the case of autarky. It can, thus, be concluded that in the long-run, high-fertility country acts as a large country under free trade and sets the terms of trade such that the world relative price converges to the level that would prevail in the large country under autarky. Under the same constant fertility rate assumption, in the long-run, the capital-labor ratio in high-fertility country (country $S$) converges to

$$\tilde{k}_s^S = \left[ n^S \left( \frac{\Phi_1^\beta}{\Phi_2} \right) \right]^{\frac{1}{\beta-1}} l = \left[ n^S \left( \frac{\Phi_1}{\Phi_2} \right) \right]^{\frac{1}{\beta-1}} \Phi_1 l \tag{6.29}$$

which is equal to the capital-labor ratio of country $S$ under autarky (see equation (3.66)). The capital-labor ratio in low-fertility country (country $N$) will converge to

$$\tilde{k}_s^N = \left( \frac{n^S}{n^N} \right) \left[ n^S \left( \frac{\Phi_1^\beta}{\Phi_2} \right) \right]^{\frac{1}{\beta-1}} l = \left( \frac{n^S}{n^N} \right) \left[ n^S \left( \frac{\Phi_1}{\Phi_2} \right) \right]^{\frac{1}{\beta-1}} \Phi_1 l \tag{6.30}$$

which is higher than the long-run capital-labor ratio of $S$ by a factor of $\frac{n^S}{n^N} > 1$. However, compared to the ratio that would prevail in $N$ under autarky, it can be easily shown that free trade lowers the long-run level of capital-labor ratio in country $N$.

**Corollary 1:** *If the long-run fertility rates are different between two otherwise identical countries, free trade in the long-run results in a decrease in capital per worker and relative price in low-fertility country and do not change the capital per worker and the relative price in high-fertility country.*

**Proof:** For the low-fertility country, using equations (3.66) and (6.30) to compare long-run autarky and free trade levels of $k^N$ one easily concludes that

$$k_s^N = k_s^N = \left(n^N\right)^{\frac{1}{\beta-1}} \left(\frac{\Phi_1}{\Phi_2}\right)^{\frac{1}{\beta-1}} \Phi_1 l > \frac{\left(n^S\right)^{\frac{\beta}{\beta-1}}}{n^N} \left(\frac{\Phi_1}{\Phi_2}\right)^{\frac{1}{\beta-1}} \Phi_1 l = \tilde{k}_s^N$$

since $\beta < 1$ and $n^N < n^S$ imply that $\left(n^N\right)^{\frac{\beta}{\beta-1}} > \left(n^S\right)^{\frac{\beta}{\beta-1}}$. For the high-fertility country, equations (3.66) and (6.29) directly imply that $k_s^S = \tilde{k}_s^S$. Similarly, using (3.61) and (6.28) $p_s^S = \left[n^S \left(\frac{\Phi_1}{\Phi_2}\right)\right]^{\frac{\beta-\alpha}{\beta-1}} = \tilde{p}_s < \left[n^N \left(\frac{\Phi_1}{\Phi_2}\right)\right]^{\frac{\beta-\alpha}{\beta-1}}$ since $n^N < n^S$ and $\beta < 1$. □

In other words, when compared to the autarky, free trade under a dynamic HO setting lowers the capital-labor ratio in the low-fertility country without changing this ratio in the high-fertility country in the long-run. This result is identical with the free trade implications of differential population growth rates demonstrated in Sayan (2005) and Jelassi (2004).

### 6.1.2 Closed-form Solutions of the Model Variables and the Long-run Implications of a Change in Fertility Rate

To derive the steady state solutions for all the model variables, one can insert the steady state solutions for the world relative price and per worker capital stocks in $S$ and $N$ given by (6.28), (6.29) and (6.30) respectively into the optimality conditions derived from profit and utility maximization problems. The following corollaries summarize the long-run implications of a change in fertility rate for world wage and rental rates, sectoral allocations of capital and labor, sectoral outputs and the amounts of consumption, in each country.

**Corollary 2:** *Under free trade, the world relative price of good 1 is decreasing in $n^S$, i.e., the fertility rate of the high-fertility country.*

**Proof:** From (6.28), $\frac{\partial \tilde{p}_s}{\partial n^S} = \left(\frac{\beta-\alpha}{\beta-1}\right)\left(\frac{\Phi_1}{\Phi_2}\right)^{\frac{\beta-\alpha}{\beta-1}} (n^S)^{\frac{1-\alpha}{\beta-1}} < 0$ since $1 > \beta > \alpha$. □

**Corollary 3:** *The common world wage rate is decreasing and the common world rental rate is increasing in the fertility rate of high-fertility country, $n^S$. Moreover, in the long-run free trade lowers (raises) the wage rate and raises (lowers) the rental rate in the low-fertility (high-fertility) country.*

**Proof:** From (3.34) and (3.33), it is clear that $\frac{\partial \tilde{w}_s}{\partial n^S} = \left[A(1-\alpha)\delta^{\alpha}\left(\frac{\beta}{\beta-\alpha}\right)\tilde{p}_s^{\frac{\alpha}{\beta-\alpha}}\right]\frac{\partial \tilde{p}_s}{\partial n^S} < 0$ and $\frac{\partial \tilde{r}_s}{\partial n^S} = \left[A\alpha\delta^{\alpha-1}\left(\frac{\beta-1}{\beta-\alpha}\right)p_s^{\frac{\alpha-1}{\beta-\alpha}}\right]\frac{\partial \tilde{p}_s}{\partial n^S} > 0$ since $\frac{\partial \tilde{p}_s}{\partial n^S} < 0$. The second part follows from Corollary 1, since $\frac{\partial w}{\partial p} > 0$ and $\frac{\partial r}{\partial p} < 0$. □

Corollary 2 and Corollary 3 state that fertility shocks in large countries are capable of changing world relative prices as well as wage and rental rates which are taken more or less given by smaller countries. In other words, the effects of demographic changes in large and industrialized countries can be passed onto small and less industrialized countries through changes in common world relative prices. Considering the case of

EU and Turkey, Kenc and Sayan (2001) establish a similar result, who argue that trade acts as a transmission channel for the effects of demographic shocks onto small countries through relative prices. They find that spillovers of the demographic shocks in EU countries may magnify the effects of Turkey's own demographic transition.

**Corollary 4:** *In the long-run,*

**i.** *the capital-labor ratio in high-fertility country $S$ is decreasing in its own steady state fertility rate $n^S$,*

**ii.** *the capital-labor ratio in low-fertility country $N$ is decreasing both in its own steady state fertility rate $n^N$ and the steady state fertility rate of country $S$, $n^S$.*

**Proof:**

**i.** $$\frac{\partial \tilde{k}_s^S}{\partial n^S} = \frac{1}{\beta - 1}\left( n^S \frac{\Phi_1{}^{\beta}}{\Phi_2} \right)^{-1} \left( \frac{\Phi_1{}^{\beta}}{\Phi_2} \right) l < 0 \text{ since } \beta < 1.$$

**ii.** $$\frac{\partial \tilde{k}_s^N}{\partial n^N} = -\left( \frac{1}{n^N} \right)^2 (n^S)^{\frac{\beta}{\beta-1}} \left( \frac{\Phi_1{}^{\beta}}{\Phi_2} \right)^{\frac{1}{\beta-1}} < 0 \text{ and}$$

$$\frac{\partial \tilde{k}_s^N}{\partial n^S} = \left( \frac{\beta}{\beta - 1} \right) \frac{1}{n^N} (n^S)^{\frac{1}{\beta-1}} \left( \frac{\Phi_1{}^{\beta}}{\Phi_2} \right)^{\frac{1}{\beta-1}} < 0 . \square$$

This corollary states that increasing birth rates in one country not only reduces the capital per worker in that country, but also reduces that in the other country.

**Corollary 5:** *In the long-run,*

**i.** *in the high-fertility country, per worker capital employed in both the labor-intensive sector (sector 1) and the capital-intensive sector (sector 2) are decreasing in $n^S$;*

**ii.** *in the low-fertility country, the effect of an increase in* $n^N$ *is positive on* $\tilde{k}_{1s}^N$ *and negative on* $\tilde{k}_{2s}^N$. *The effect of an increase in* $n^S$ *on* $\tilde{k}_{1s}^N$ *and* $\tilde{k}_{2s}^N$ *is ambiguous;*

**iii.** *in country* $N$, *free trade lowers capital per worker employed in sector 1 whereas the effect of free trade on capital per worker employed in sector 2 is ambiguous. In the long-run, free trade does not change sectoral capital stocks per worker in country* $S$.

**Proof:**

**i.** By (3.29), $\tilde{k}_{1s}^S = \dfrac{\delta l\left(\varepsilon - \Phi_1\right)}{\varepsilon - \delta}\left(n^S \dfrac{\Phi_1}{\Phi_2}\right)^{\frac{1}{\beta-1}}$, which is clearly decreasing in $n^S$. By (3.30), $\tilde{k}_{2s}^S = \dfrac{\varepsilon l\left(\Phi_1 - \delta\right)}{\varepsilon - \delta}\left(n^S \dfrac{\Phi_1}{\Phi_2}\right)^{\frac{1}{\beta-1}}$, which is also decreasing in $n^S$.

**ii.** Since by (3.29), $\tilde{k}_{1s}^N = \dfrac{\delta l}{\varepsilon - \delta}\left(\varepsilon - \dfrac{n^S}{n^N}\Phi_1\right)\left(n^S \dfrac{\Phi_1}{\Phi_2}\right)^{\frac{1}{\beta-1}}$, we have

$\dfrac{\partial \tilde{k}_{1s}^N}{\partial n^N} = \dfrac{\delta \Phi_1 l}{\varepsilon - \delta}\dfrac{n^S}{(n^N)^2}\left(n^S \dfrac{\Phi_1}{\Phi_2}\right)^{\frac{1}{\beta-1}} > 0$. Since by (3.30),

$\tilde{k}_{2s}^N = \dfrac{\varepsilon l}{\varepsilon - \delta}\left(\dfrac{n^S}{n^N}\Phi_1 - \delta\right)\left(n^S \dfrac{\Phi_1}{\Phi_2}\right)^{\frac{1}{\beta-1}}$, we have

$\dfrac{\partial \tilde{k}_{2s}^N}{\partial n^N} = -\dfrac{\varepsilon \Phi_1 l}{\varepsilon - \delta}\dfrac{n^S}{(n^N)^2}\left(n^S \dfrac{\Phi_1}{\Phi_2}\right)^{\frac{1}{\beta-1}} < 0$.

On the other hand,

$$\frac{\partial \tilde{k}_{1s}^N}{\partial n^S} = \left(\frac{1}{1-\beta}\right)\left(\frac{\delta l}{\varepsilon - \delta}\right)(n^S)^{\frac{1}{\beta-1}}\left(\frac{\beta \Phi_1}{n^N} - \frac{\varepsilon}{n^S}\right)\left(\frac{\Phi_1}{\Phi_2}\right)^{\frac{1}{\beta-1}}$$

and

$$\frac{\partial \tilde{k}_{2s}^N}{\partial n^S} = \left(\frac{1}{\beta-1}\right)\left(\frac{\varepsilon l}{\varepsilon-\delta}\right)(n^S)^{\frac{1}{\beta-1}}\left(\frac{\beta\Phi_1}{n^N} - \frac{\delta}{n^S}\right)\left(\frac{\Phi_1}{\Phi_2}\right)^{\frac{1}{\beta-1}}$$

These imply that

$$\frac{\partial \tilde{k}_{1s}^N}{\partial n^S} \begin{cases} \geq 0 \text{ if } \dfrac{n^N}{n^S} \leq \dfrac{\beta\Phi_1}{\varepsilon} \\ < 0 \text{ if } otherwise \end{cases}$$

and

$$\frac{\partial \tilde{k}_{2s}^N}{\partial n^S} \begin{cases} \geq 0 \text{ if } \dfrac{n^N}{n^S} \leq \dfrac{\beta\Phi_1}{\delta} \\ < 0 \text{ if } otherwise \end{cases}$$

**iii.** By (3.29) and (6.30)

$$\tilde{k}_{1s}^N = -\frac{\delta}{\varepsilon-\delta}\tilde{k}_s^N + \frac{\delta\varepsilon}{\varepsilon-\delta} l\tilde{p}_s^{\frac{1}{\beta-\alpha}} = \frac{\delta}{\varepsilon-\delta} l\left(n^S \frac{\Phi_1}{\Phi_2}\right)^{\frac{1}{\beta-1}}\left(\varepsilon - \frac{n^S}{n^N}\Phi_1\right)$$

and from (3.67), we have

$$k_{1s}^N = \frac{\delta}{\varepsilon-\delta} l\left(n^N \frac{\Phi_1}{\Phi_2}\right)^{\frac{1}{\beta-1}}\left(\varepsilon - \Phi_1\right)$$

Since $\Phi_1 > 0$ and $n^S > n^N$, $k_{1s}^N > \tilde{k}_{1s}^N$. Similarly,

$$\tilde{k}_{2s}^N = \frac{\varepsilon}{\varepsilon-\delta} l\left(n^S \frac{\Phi_1}{\Phi_2}\right)^{\frac{1}{\beta-1}}\left(\frac{n^S}{n^N}\Phi_1 - \delta\right)$$

and

$$k_{2s}^N = \frac{\varepsilon}{\varepsilon-\delta} l\left(n^N \frac{\Phi_1}{\Phi_2}\right)^{\frac{1}{\beta-1}}\left(\Phi_1 - \delta\right)$$

These imply that $k_{2s}^N > \tilde{k}_{2s}^N$ if and only if $\dfrac{n^S}{n^N} > \left[\dfrac{\left(\dfrac{n^S}{n^N}\Phi_1\right) - \delta}{\Phi_1 - \delta}\right]^{1-\beta}$.

For country $S$, since steady state relative prices and capital per workers under autarky and trade are equal by Corollary 1, it directly follows that $k_{1s}^S = \tilde{k}_{1s}^S$ and $k_{2s}^S = \tilde{k}_{2s}^S$. □

**Corollary 6:** *In the long-run,*

**i.** *the sectoral labor allocation in high-fertility country is not affected by a change in the fertility rate;*

**ii.** *in low-fertility country, the effect of an increase in* $n^N$ *is positive on* $\tilde{l}_{1s}^N$ *and negative on* $\tilde{l}_{2s}^N$*, whereas the effect of an increase in* $n^S$ *is negative on* $\tilde{l}_{1s}^N$ *and positive on* $\tilde{l}_{2s}^N$*.*

**iii.** *free trade lowers* $l_1^N$ *and raises* $l_2^N$ *with respect to autarky, whereas* $l_1^S$ *and* $l_2^S$ *do not change.*

**Proof:**

**i.** By (3.31) and (3.32), $\tilde{l}_{1s}^S = \frac{(\varepsilon - \Phi_1)}{\varepsilon - \delta} l$ and $\tilde{l}_{2s}^S = \frac{(\Phi_1 - \delta)}{\varepsilon - \delta} l$. Hence, the sectoral allocation of labor is independent of fertility rates.

**ii.** By (3.31) and (3.32), $\tilde{l}_{1s}^N = \frac{\left(\varepsilon - \frac{n^S}{n^N}\Phi_1\right)}{\varepsilon - \delta} l$ and $\tilde{l}_{2s}^N = \frac{\left(\frac{n^S}{n^N}\Phi_1 - \delta\right)}{\varepsilon - \delta} l$. Hence,

$\frac{\partial \tilde{l}_{1s}^N}{\partial n^N} > 0$ and $\frac{\partial \tilde{l}_{1s}^N}{\partial n^S} < 0$, while $\frac{\partial \tilde{l}_{2s}^N}{\partial n^N} < 0$ and $\frac{\partial \tilde{l}_{2s}^N}{\partial n^S} > 0$ obviously.

**iii.** $\tilde{l}_{1s}^N = \frac{1}{\varepsilon - \delta} l \left(\varepsilon - \frac{n^S}{n^N}\Phi_1\right)$ and $l_{1s}^N = \frac{1}{\varepsilon - \delta} l(\varepsilon - \Phi_1)$. Since $n^S > n^N$, $l_{1s}^N > \tilde{l}_{1s}^N$.

But then $\tilde{l}_{1s}^N + \tilde{l}_{2s}^N = 1$ implies $l_{2s}^N < \tilde{l}_{2s}^N$. □

It is worth pointing out that a corner solution is a possibility for the low-fertility country. At the steady state, this country may completely specialize in the production of good 2 and export good 1 from high-fertility country to meet the whole domestic demand. This may arise if and only if $\tilde{l}_{1s}^{N}=0$, i.e., $\frac{\varepsilon}{\Phi_1}=\frac{n^S}{n^N}$. Therefore, to guarantee interior solutions in the numeric exercises analyzed in Section 6.3, the values of $n^N$ and $n^S$ are picked such that $\frac{n^S}{n^N}<\frac{\varepsilon}{\Phi_1}$ is satisfied.

**Corollary 7:** *In the long-run,*

**i.** *per worker outputs of both sectors in high-fertility country are decreasing in fertility rate $n^S$;*

**ii.** *in low-fertility country, per worker output of good 1 is increasing and the output of good 2 is decreasing in $n^N$, while the effect of a change in $n^S$ on $\tilde{x}_{1s}^{N}$ and $\tilde{x}_{2s}^{N}$ is ambiguous.*

**iii.** *in country $N$, free trade lowers the output per worker of sector 1, whereas its effect on the output per worker of sector 2 is ambiguous. Steady state output levels in $S$ do not change in response to the introduction of trade.*

**Proof:**

**i.** Using the expressions in (3.35) and (3.36) and the results in Corollary 2 and Corollary 6 (i), one can write

$$\frac{\partial \tilde{x}_{1s}^{S}}{\partial n^S}=A\delta^{\alpha}\left[\frac{\partial \tilde{l}_{1s}^{S}}{\partial n^S}+\frac{\alpha}{\beta-\alpha}p_t^{\frac{2\alpha-\beta}{\beta-\alpha}}\frac{\partial \tilde{p}_s}{\partial n^S}\right]=A\delta^{\alpha}\frac{\alpha}{\beta-\alpha}p_t^{\frac{2\alpha-\beta}{\beta-\alpha}}\frac{\partial \tilde{p}_s}{\partial n^S}<0. \text{ Similarly,}$$

$$\frac{\partial \tilde{x}_{2s}^{S}}{\partial n^S}=A\varepsilon^{\beta}\left[\frac{\partial \tilde{l}_{2s}^{S}}{\partial n^S}+\frac{\beta}{\beta-\alpha}p_t^{\frac{\alpha}{\beta-\alpha}}\frac{\partial \tilde{p}_s}{\partial n^S}\right]=A\varepsilon^{\beta}\frac{\beta}{\beta-\alpha}p_t^{\frac{\alpha}{\beta-\alpha}}\frac{\partial \tilde{p}_s}{\partial n^S}<0.$$

**ii.** By (3.37) and (3.38),

$$\tilde{x}_{1s}^{N}=A\delta^{\alpha}\frac{\left(\varepsilon-\frac{n^S}{n^N}\Phi_1\right)l}{\varepsilon-\delta}\left(n^S\frac{\Phi_1}{\Phi_2}\right)^{\frac{\alpha}{\beta-1}}$$

and

$$\tilde{x}_{2s}^N = A\varepsilon^\beta \frac{\left(\frac{n^S}{n^N}\Phi_1 - \delta\right)l}{\varepsilon - \delta}\left(n^S \frac{\Phi_1}{\Phi_2}\right)^{\frac{\beta}{\beta-1}}$$

From these expressions it follows that $\frac{\partial \tilde{x}_{1s}^N}{\partial n^N} > 0$ and $\frac{\partial \tilde{x}_{2s}^N}{\partial n^N} < 0$. After taking the derivative of sectoral per worker outputs with respect to $n^S$ and doing the necessary manipulations, it can be shown that

$$\frac{\partial \tilde{x}_{1s}^N}{\partial n^S}\begin{cases} \geq 0 \text{ if } \frac{n^N}{n^S} \geq \left|\frac{\Phi_1}{\varepsilon}\left(\frac{\alpha+\beta-1}{\alpha}\right)\right| \\ < 0 \text{ if } \textit{otherwise} \end{cases}$$

and

$$\frac{\partial \tilde{x}_{2s}^N}{\partial n^S}\begin{cases} \geq 0 \text{ if } \frac{n^N}{n^S} \leq \left|\frac{\Phi_1}{\delta}\left(\frac{2\beta-1}{\beta}\right)\right| \\ < 0 \text{ if } \textit{otherwise} \end{cases}$$

**iii.** From (3.35)

$$\tilde{x}_{1s}^N = A\delta^\alpha \tilde{l}_{1s}^N \tilde{p}_s^{\frac{\alpha}{\beta-\alpha}} \text{ and } x_{1s}^N = A\delta^\alpha l_{1s}^N p_s^{N\frac{\alpha}{\beta-\alpha}}$$

In Corollary 1 and 6, it is already established that $\tilde{p}_s < p_s^N$ and $\tilde{l}_{1s}^N < l_{1s}^N$. Hence $x_{1s}^N > \tilde{x}_{1s}^N$. As for the output of sector 2, using (3.36) and calculating steady state values under autarky and free trade, we conclude

$$x_{2s}^N > \tilde{x}_{2s}^N \text{ iff } \frac{n^S}{n^N} > \frac{\left(\frac{n^S}{n^N}\Phi_1\right) - \delta}{\Phi_1 - \delta}$$

The last part directly follows from the fact that $\tilde{p}_s = p_s^S$ and $\tilde{k}_s^S = k_s^S$.

□

**Corollary 8:** *In the long-run,*

**i.** *in high-fertility country, consumption per child* $\tilde{c}_{1s}^{S}$ *and consumption per adult* $\tilde{c}_{2ys}^{S}$ *are decreasing in* $n^{S}$*, while the effect of an increase in* $n^{S}$ *on consumption per old* $\tilde{c}_{2os}^{S}$ *is ambiguous;*

**ii.** *in low-fertility country, consumption per child* $\tilde{c}_{1s}^{N}$ *is decreasing in* $n^{N}$*; both consumption per adult* $\tilde{c}_{2ys}^{N}$ *and consumption per child* $\tilde{c}_{1s}^{N}$ *are decreasing in* $n^{S}$*, but the effect of* $n^{S}$ *on consumption per old* $\tilde{c}_{2os}^{N}$ *is ambiguous;*

**iii.** *in country* $N$*, free trade lowers the amount of consumption per child and per adult, while its effect on old consumption is ambiguous.*

**Proof:**

**i.-ii.:** By (3.11), (3.12) and from the factor price equalization assumption under trade, adult and old consumptions in $N$ are equal to adult and old consumptions in $S$ respectively. Therefore, we have

$$\frac{\partial \tilde{c}_{2ys}^{S}}{\partial n^{S}} = \frac{\partial \tilde{c}_{2ys}^{N}}{\partial n^{S}} = A\mu(1-\theta)(1-\alpha)\delta^{\alpha} l\left(\frac{\beta}{\beta-1}\right)\left(\frac{\Phi_1}{\Phi_2}\right)^{\frac{\beta}{\beta-1}}(n^{S})^{\frac{1}{\beta-1}}$$

and

$$\frac{\partial \tilde{c}_{2os}^{S}}{\partial n^{S}} = \frac{\partial \tilde{c}_{2os}^{N}}{\partial n^{S}} = A(1-\mu)(1-\alpha)\delta^{\alpha} l\left(\frac{1}{\beta-1}\right)\left(\frac{\Phi_1}{\Phi_2}\right)^{\frac{\beta}{\beta-1}}(n^{S})^{\frac{1}{\beta-1}}\left[\beta + A\alpha\delta^{\alpha-1}(2\beta-1)\left(\frac{\Phi_1}{\Phi_2}\right)n^{S}\right]$$

From the first equation, it is clear that $\frac{\partial \tilde{c}_{2ys}^{S}}{\partial n^{S}} = \frac{\partial \tilde{c}_{2ys}^{N}}{\partial n^{S}} < 0$ and from the second one, it follows that

$$\frac{\partial \tilde{c}_{2os}^{S}}{\partial n^{S}} = \frac{\partial \tilde{c}_{2os}^{N}}{\partial n^{S}} \begin{cases} \geq 0 \text{ if } n^{S} \leq \left|\frac{\beta}{(1-2\beta)A\alpha\delta^{\alpha-1}}\left(\frac{\Phi_2}{\Phi_1}\right)\right| \\ < 0 \text{ if } \textit{otherwise} \end{cases}$$

By (3.10), from the derivatives of per child consumption with respect to $n^S$ in countries $S$ and $N$, we have

$$\frac{\partial \tilde{c}_{1s}^S}{\partial n^S} = A\mu\theta(1-\alpha)\delta^\alpha l\left(\frac{1+\alpha-\beta}{\beta-1}\right)\left(\frac{\Phi_1}{\Phi_2}\right)^{\frac{\alpha}{\beta-1}}(n^S)^{\frac{2+\alpha-2\beta}{\beta-1}} < 0$$

and

$$\frac{\partial \tilde{c}_{1s}^N}{\partial n^S} = A\mu\theta(1-\alpha)\delta^\alpha l\left(\frac{\alpha}{\beta-1}\right)\left(\frac{\Phi_1}{\Phi_2}\right)^{\frac{\alpha}{\beta-1}}\frac{(n^S)^{\frac{1+\alpha-\beta}{\beta-1}}}{n^N} < 0$$

Finally, it is obvious again by (3.10) that $\frac{\partial \tilde{c}_{1s}^N}{\partial n^N} < 0$.

**iii.** Using (3.10), we have

$$\tilde{c}_{1s}^N = \frac{A\mu\theta(1-\alpha)\delta^\alpha l}{n^N}\left(n^S\frac{\Phi_1}{\Phi_2}\right)^{\frac{\alpha}{\beta-1}} \text{ and } c_{1s}^N = \frac{A\mu\theta(1-\alpha)\delta^\alpha l}{n^N}\left(n^N\frac{\Phi_1}{\Phi_2}\right)^{\frac{\alpha}{\beta-1}}.$$

Similarly, using (3.11), we have

$$\tilde{c}_{2ys}^N = A\mu(1-\theta)(1-\alpha)\delta^\alpha l\left(n^S\frac{\Phi_1}{\Phi_2}\right)^{\frac{\beta}{\beta-1}} \quad \text{and}$$

$$\tilde{c}_{2ys}^N = A\mu(1-\theta)(1-\alpha)\delta^\alpha l\left(n^N\frac{\Phi_1}{\Phi_2}\right)^{\frac{\beta}{\beta-1}}.$$

Then, since $n^S > n^N$ and $\beta < 1$, it follows that $c_{1s}^N > \tilde{c}_{1s}^N$ and $c_{2ys}^N > \tilde{c}_{2ys}^N$. Finally, using (3.12), we have

$$\tilde{c}_{2os}^N = \left(\Phi_2 + A\alpha\delta^{\alpha-1}n^S\Phi_1\right)l\left(n^S\frac{\Phi_1}{\Phi_2}\right)^{\frac{\beta}{\beta-1}} \quad \text{and}$$

$$c_{2os}^N = \left(\Phi_2 + A\alpha\delta^{\alpha-1}n^N\Phi_1\right)l\left(n^N\frac{\Phi_1}{\Phi_2}\right)^{\frac{\beta}{\beta-1}}.$$

This, in turn, implies that $c_{20s}^N > \tilde{c}_{2os}^N$ iff $\frac{n^S}{n^N} > \frac{\Phi_2 + A\alpha\delta^{\alpha-1}n^S\Phi_1}{\Phi_2 + A\alpha\delta^{\alpha-1}n^N\Phi_1}$. □

These results suggest that in the long-run, higher fertility in country $S$ has a welfare reducing effect for both economies, whereas an increase in the fertility rate of country $N$ has no effect on the welfare of $S$ as long as $S$ remains a large country relative to $N$, i.e., $n^S > n^N$ continues to hold. As for the welfare implications of free trade between two countries with different fertility rates, one cannot reach a general conclusion due to the ambiguity in the effect of free trade on olds' consumption. Although free trade clearly lowers the welfare of child and adult generations at the steady state, the old generation may be better off as well as worse off. Whether a possible improvement in the amount of old consumption can dominate the life-time utility loss due to lower child and adult consumption depends on the model parameters as well as on $n^S$ and $n^N$.

## 6.2 The Mortality Anticipation Model under Free Trade

This section introduces the free trade extension of the 'anticipation' model presented in Chapter 5. In this model, agents has the capacity to anticipate their probability of death at the end of the adulthood and to decide saving and consumption accordingly. Under free trade, the market clearance and factor price equalization conditions given for the basic model through the equations (6.1)-(6.8) also apply to the 'anticipation' model. Due to the difference in saving rates across the two models, the capital market clearance conditions of each country under the 'anticipation' model are modified as

$$\tilde{k}_{t+1}^{i} = \frac{A\left(1-\alpha\right)(mps_t^i)l\delta^{\alpha}\,\tilde{p}_t^{\frac{\beta}{\beta-\alpha}}}{n_t^i} \tag{6.31}$$

for each $i \in \{N, S\}$.

The analytical solution of the free trade equilibrium under the assumption of complete survival into old age is presented below. The numerical solutions of the model under positive mortality and the analysis of various demographic scenarios are presented in Section 6.3.

### 6.2.1 The Dynamic Equilibrium under Full Certainty of Survival into Old Age

Once again confining ourselves to the assumption $\lambda_t^N = \lambda_t^S = 1,\ \forall t$, the closed form solutions of steady state variables can be obtained. This assumption amounts to imposing the restriction that the marginal propensities to consume good 1 and good 2 in the first period ($m_1$ and $m_{2y}$) and the marginal propensity to save ($mps$) in both countries are constant and equal to each other. The global market clearance condition for good 1 is given by

$$N_t^S\tilde{x}_{1t}^S + N_t^N\tilde{x}_{1t}^N = N_t^S n_t^S \tilde{c}_{1t}^S + N_t^N n_t^N \tilde{c}_{1t}^N \tag{6.32}$$

Inserting the expressions for $x_{1t}^{i}$ and $c_{1t}^{i}$ given in (5.6) and (5.5), the condition in (6.32) implies the following relationship between the price ratio and per worker capital stocks in each country:

$$\tilde{p}_t = \left\{ \frac{\tilde{N}_t l \left[ \varepsilon\left(1-\left(1-\alpha\right)m_1\right)+\delta\left(1-\alpha\right)m_1 \right]}{N_t^S \tilde{k}_t^S + N_t^N \tilde{k}_t^N} \right\}^{\alpha-\beta} = \left[ \frac{\phi_1 \tilde{N}_t l}{N_t^S \tilde{k}_t^S + N_t^N \tilde{k}_t^N} \right]^{\alpha-\beta} \tag{6.33}$$

where $m_1 = \dfrac{\theta\left(1+\rho\right)}{2+\rho}$. Using the saving-investment relationships $n_t^N k_{t+1}^N = s_t^N$ and $n_t^S k_{t+1}^S = s_t^S$, and using the fact that $s_t^i = A\left(1-\alpha\right)(mps)\delta^{\alpha}\, \tilde{p}_t^{\frac{\beta}{\beta-\alpha}}$, $i \in \left\{N, S\right\}$, the sum of country wide capital stocks in each country is written as

$$N_{t+1}^S \tilde{k}_{t+1}^S + N_{t+1}^N \tilde{k}_{t+1}^N = A\left(1-\alpha\right)(mps)\delta^{\alpha}\, \tilde{p}_t^{\frac{\beta}{\beta-\alpha}} \tilde{N}_t l \tag{6.34}$$

where $\tilde{N}_t = N_t^S + N_t^N$ and $mps = \dfrac{1}{2+\rho}$ as before. Writing (6.33) for one period ahead and inserting (6.34) in the denominator, the following difference equation for the relative price is obtained

$$\frac{\tilde{p}_{t+1}}{\tilde{p}_t^{\beta}} = \left( \left(1+g_{t+1}\right) \frac{\phi_1}{\phi_2} \right)^{\alpha-\beta} \tag{6.35}$$

where $1+g_{t+1} = \dfrac{\tilde{N}_{t+1}}{\tilde{N}_t}$ and $\phi_1$ and $\phi_2$ are defined as in Section 5.1. Alternatively, rewriting (6.33) as

$$N_t^S \tilde{k}_t^S + N_t^N \tilde{k}_t^N = \phi_1 \tilde{N}_t l \tilde{p}_t^{\frac{1}{\beta-\alpha}} \tag{6.36}$$

and plugging $\tilde{p}_t^{\frac{1}{\beta-\alpha}} = \left[ \frac{N_{t+1}^S \tilde{k}_{t+1}^S + N_{t+1}^N \tilde{k}_{t+1}^N}{\phi_2 \tilde{N}_t l} \right]^{\frac{1}{\beta}}$, which is derived from (6.34), in (6.36), the law of motion for capital stock implied by the inter-temporal equilibrium is written as

$$\frac{\tilde{K}_{t+1}}{\tilde{K}_t^{\beta}} = \frac{\phi_2 \tilde{N}_t l}{\left(\phi_1 \tilde{N}_t l\right)^{\beta}} \tag{6.37}$$

where $\tilde{K}_t = K_t^S + K_t^N$ denotes the total capital stock in both countries. If $\tilde{k}_t$ is defined as the average capital stock per worker in both countries (or global capital-labor ratio), then it can be denoted by $\tilde{k}_t = \frac{\tilde{K}_t}{\tilde{N}_t} = \frac{N_t^S \tilde{k}_t^S + N_t^N \tilde{k}_t^N}{N_t^N + N_t^S} = \gamma_t^S \tilde{k}_t^S + \gamma_t^N \tilde{k}_t^N$, where $\gamma_t^N$ and $\gamma_t^S$ are the ratios of worker populations in country $N$ and country $S$ to the world's worker population at time $t$ respectively. Then, (6.37) can be rewritten in terms of average capital stock per worker as

$$\frac{\tilde{k}_{t+1}}{\tilde{k}_t^{\beta}} = \frac{\phi_2}{\left(1+g_{t+1}\right)\phi_1^{\beta}} l^{1-\beta} \tag{6.38}$$

At the steady state $\tilde{p}_{t+1} = \tilde{p}_t = \tilde{p}_s$, $\tilde{k}_{t+1} = \tilde{k}_t = \tilde{k}_s$ and $n_t^i = n_{t+1}^i = n^i$ for $i \in \{N, S\}$. Thus, from (6.35) and (6.38) the steady state price ratio and average capital stock per worker are equal to

$$\tilde{p}_s = \left[ \left(1+g_s\right) \frac{\phi_1}{\phi_2} \right]^{\frac{\alpha-\beta}{1-\beta}} \tag{6.39}$$

and

$$\tilde{k}_s = \left[ \left(1+g_s\right) \frac{\phi_1^{\beta}}{\phi_2} \right]^{\frac{1}{\beta-1}} l = \left[ \left(1+g_s\right) \frac{\phi_1}{\phi_2} \right]^{\frac{1}{\beta-1}} \phi_1 l \tag{6.40}$$

Once again using the saving-investment relationships $\tilde{k}_s^i = \frac{\tilde{s}_t^i}{n^i} = \frac{A(1-\alpha)(mps)\delta^\alpha l}{n^i} \tilde{p}_s^{\frac{\beta}{\beta-\alpha}}$ for each country, the country specific capital-labor ratios are written as

$$\tilde{k}_s^N = \frac{(1+g_s)^{\frac{\beta}{\beta-1}}}{n^N}\left(\frac{\phi_1^\beta}{\phi_2}\right)^{\frac{1}{\beta-1}} l \tag{6.41}$$

and

$$\tilde{k}_s^S = \frac{(1+g_s)^{\frac{\beta}{\beta-1}}}{n^S}\left(\frac{\phi_1^\beta}{\phi_2}\right)^{\frac{1}{\beta-1}} l \tag{6.42}$$

$(1+g_s)$ can be expressed by $1+g_s = \lim_{t\to\infty}(1+g_t) = \lim_{t\to\infty}\left(\gamma_t^N n_t^N + \gamma_t^S n_t^S\right)$, where

$\gamma_t^S = \frac{N_t^S}{N_t^S + N_t^N}$ and $\gamma_t^N = \frac{N_t^N}{N_t^S + N_t^N}$ are the working population shares of $S$ and $N$.

Hence, if $n^S > n^N$ holds at the steady state, then $\lim_{t\to\infty}\gamma_t^S = 1$ and $\lim_{t\to\infty}\gamma_s^N = 0$, implying that $(1+g_s) = n^S$ as before. Therefore, (6.39) and (6.40) can be replaced by

$$\tilde{p}_s = \left[n^S \frac{\phi_1}{\phi_2}\right]^{\frac{\alpha-\beta}{1-\beta}} \tag{6.43}$$

and

$$\tilde{k}_s = \left[n^S \frac{\phi_1}{\phi_2}\right]^{\frac{1}{\beta-1}} \phi_1 l \tag{6.44}$$

Similarly, (6.41) and (6.42) become

$$\tilde{k}_s^N = \frac{n^S}{n^N}\left(n^S \frac{\phi_1}{\phi_2}\right)^{\frac{1}{\beta-1}} \phi_1 l \tag{6.45}$$

and

$$\tilde{k}_s^S = \left(n^S \frac{\phi_1}{\phi_2}\right)^{\frac{1}{\beta-1}} \phi_1 l \tag{6.46}$$

**Corollary 9:** *In the long-run, the world relative price* $\tilde{p}_s$*, the world per worker capital stock, and country specific per worker capital stocks are all decreasing in the fertility rate* $n^S$ *of high-fertility country.*

**Proof:** Taking the derivatives of (6.43)-(6.46) with respect to $n^S$ proves the corollary since $\beta < 1$.

### 6.2.2 The Closed-form Solutions of the Model Variables and the Long-run Effects of a Change in Fertility Rate

Comparing the steady state world relative price and country specific per worker capitals under the basic model and the anticipation model given by (6.28)-(6.30) and by (6.43), (6.45) and (6.46) respectively, one notices that these expressions are identical except that in the anticipation model, $\Phi_1$ and $\Phi_2$ of the basic model are replaced by $\phi_1$ and $\phi_2$. Since all these parameters are positive, and $\Phi_1, \phi_1$ satisfy $\varepsilon > \Phi_1 > \delta$ and $\varepsilon > \phi_1 > \delta$, the effects of a change in $n^S$ or $n^N$ on all the other variables are qualitatively identical across the two models. Moreover, the implications of free trade with respect to autarky are also identical across the two models. Therefore, unlike in the case of the basic model, here the impacts of a change in $n^S$ and $n^N$ on the steady state values given in (6.47)-(6.57) will not be explicitly demonstrated through corollaries, but will only be briefly summarized to avoid unnecessary repetition.

In line with the optimal allocation rules of capital stock between the two sectors given in Chapter 3,

$$\tilde{k}_{1s}^{i} = \frac{\delta l}{\varepsilon - \delta}\left(n^{S}\frac{\phi_1}{\phi_2}\right)^{\frac{1}{\beta-1}}\left[\varepsilon - \frac{n^{S}}{n^{i}}\phi_1\right] \tag{6.47}$$

and

$$\tilde{k}_{2s}^{i} = \frac{\varepsilon l}{\varepsilon - \delta}\left(n^{S}\frac{\phi_1}{\phi_2}\right)^{\frac{1}{\beta-1}}\left[\frac{n^{S}}{n^{i}}\phi_1 - \delta\right] \tag{6.48}$$

for each $i \in \{N, S\}$.

Similar to the basic model, long-run sectoral capital stocks per worker in high-fertility country $S$ are negatively related to $n^{S}$. In the low-fertility country, $\tilde{k}_{1s}^{N}$ is positively related to $n^{N}$, but the effect of $n^{S}$ on $\tilde{k}_{1s}^{N}$ and $\tilde{k}_{2s}^{N}$ are ambiguous. In particular,

$$\frac{\partial \tilde{k}_{1s}^{N}}{\partial n^{S}}\begin{cases} \geq 0 & \text{if } \dfrac{n^{S}}{n^{N}} \geq \dfrac{\varepsilon}{\beta\phi_1} \\ < 0 & \text{if } otherwise \end{cases}$$

and

$$\frac{\partial \tilde{k}_{2s}^{N}}{\partial n^{S}}\begin{cases} \geq 0 & \text{if } \dfrac{n^{S}}{n^{N}} \leq \dfrac{\delta}{\beta\phi_1} \\ < 0 & \text{if } otherwise \end{cases}$$

Inserting the steady state per worker capitals in (6.41) and (6.42), and the steady state level of world relative price given by (6.39), the optimal labor demand equation of each sector at the steady state becomes

$$\tilde{l}_{1s}^{i} = \frac{l}{\varepsilon - \delta}\left(\varepsilon - \frac{n^{S}}{n^{i}}\phi_1\right) \tag{6.49}$$

and

$$\tilde{l}_{2s}^{i} = \frac{l}{\varepsilon - \delta}\left(\frac{n^{S}}{n^{i}}\phi_1 - \delta\right) \tag{6.50}$$

for each $i \in \{N, S\}$. Similar to the basic model, $\frac{\partial \tilde{l}_{1s}^S}{\partial n^S} = \frac{\partial \tilde{l}_{2s}^S}{\partial n^S} = 0$, $\frac{\partial \tilde{l}_{1s}^N}{\partial n^S} < 0$ and $\frac{\partial \tilde{l}_{2s}^N}{\partial n^S} > 0$. Like in the case of the basic model, certain steady state levels of fertility rate in $N$ and $S$ may lead to corner solutions for the long-run trade equilibrium of the anticipation model as well. More precisely, the low-fertility country may completely specialize in the production of good 2, and hence $\tilde{l}_{1s}^N$ and $\tilde{k}_{1s}^N$ may become zero unless $\frac{\varepsilon}{\phi_1} > \frac{n^S}{n^N}$.

The steady state wage and rental rates are given by

$$\tilde{w}_s = A(1-\alpha)\delta^\alpha \left( n^S \frac{\phi_1}{\phi_2} \right)^{\frac{\beta}{\beta-1}} \tag{6.51}$$

and

$$\tilde{r}_s = A\alpha\delta^{\alpha-1} \left( n^S \frac{\phi_1}{\phi_2} \right) \tag{6.52}$$

where the wage rate is decreasing and the rental rate is increasing in $n^S$. Per worker outputs of each sector are given by

$$\tilde{x}_{1s}^i = A \frac{\delta^\alpha l}{\varepsilon - \delta} \left( \varepsilon - \frac{n^S}{n^i} \phi_1 \right) \left( n^S \frac{\phi_1}{\phi_2} \right)^{\frac{\alpha}{\beta-1}} \tag{6.53}$$

and

$$\tilde{x}_{2s}^i = A \frac{\varepsilon^\beta l}{\varepsilon - \delta} \left( \frac{n^S}{n^i} \phi_1 - \delta \right) \left( n^S \frac{\phi_1}{\phi_2} \right)^{\frac{\beta}{\beta-1}} \tag{6.54}$$

where $\frac{\partial \tilde{x}_{1s}^S}{\partial n^S} < 0$ and $\frac{\partial \tilde{x}_{2s}^S}{\partial n^S} < 0$, while for country $N$, $\frac{\partial \tilde{x}_{1s}^N}{\partial n^N} > 0$, $\frac{\partial \tilde{x}_{1s}^N}{\partial n^N} < 0$,

$$\frac{\partial \tilde{x}_{1s}^N}{\partial n^S}\begin{cases} \geq 0 \text{ if } \dfrac{n^S}{n^N} \geq \left|\dfrac{\alpha\varepsilon}{(\alpha+\beta-1)\phi_1}\right| \\ < 0 \text{ if } otherwise \end{cases}$$

and

$$\frac{\partial \tilde{x}_{2s}^N}{\partial n^S}\begin{cases} \geq 0 \text{ if } \dfrac{n^S}{n^N} \leq \left|\dfrac{\beta\delta}{(2\beta-1)\phi_1}\right|. \\ < 0 \text{ if } otherwise \end{cases}$$

Finally, the steady state consumption levels are given by

$$\tilde{c}_{1s}^i = \frac{Am_1(1-\alpha)\delta^\alpha l}{n^i}\left(n^S \frac{\phi_1}{\phi_2}\right)^{\frac{\alpha}{\beta-1}} \tag{6.55}$$

$$\tilde{c}_{2ys}^i = Am_{2y}(1-\alpha)\delta^\alpha l\left(n^S \frac{\phi_1}{\phi_2}\right)^{\frac{\beta}{\beta-1}} \tag{6.56}$$

and

$$\tilde{c}_{2os}^i = A(mps)(1-\alpha)\delta^\alpha l\left\{\left(n^S \frac{\phi_1}{\phi_2}\right)^{\frac{\beta}{\beta-1}} + A\alpha\delta^{\alpha-1}\left(n^S \frac{\phi_1}{\phi_2}\right)^{\frac{2\beta-1}{\beta-1}}\right\} \tag{6.57}$$

These imply that $\frac{\partial \tilde{c}_{1s}^S}{\partial n^S} < 0$, $\frac{\partial \tilde{c}_{2ys}^S}{\partial n^S} = \frac{\partial \tilde{c}_{2ys}^N}{\partial n^S} < 0$, $\frac{\partial \tilde{c}_{1s}^N}{\partial n^N} < 0$, $\frac{\partial \tilde{c}_{1s}^N}{\partial n^S} < 0$ and

$$\frac{\partial \tilde{c}_{2os}^S}{\partial n^S} = \frac{\partial \tilde{c}_{2os}^N}{\partial n^S}\begin{cases} \geq 0 \text{ if } n^S \leq \left|\dfrac{\beta}{(1-2\beta)A\alpha\delta^{\alpha-1}}\dfrac{\phi_2}{\phi_1}\right|. \\ < 0 \text{ if } otherwise \end{cases}$$

Similar to the case of autarky, the long-run free trade results given above and in part 6.1.2 suggest that under the assumption of complete survival into old age, the two models behave in the same way, since the only critical difference between the two models is the way mortality rate is incorporated to these models. The next section presents several simulation exercises to analyze the impact of demographic shocks – especially shocks to mortality rate– on high- and low-fertility countries and to pindown the qualitative differences that emerge between the responses of each model when assumption of complete survival into old age is relaxed.

## 6.3 Numeric Solutions of the Basic and Anticipation Models and Demographic Scenarios under Free Trade

The steady state equilibria of the above free trade models are not analytically solvable if the adult mortality is positive. It is therefore necessary to rely on numeric solution of these models under a given set of parameters and initial values. The numeric solutions of these models are initialized by per worker sectoral capital stocks in countries $S$ and $N$ for $t=1$. Given $\tilde{k}_{11}^S, \tilde{k}_{11}^N, \tilde{k}_{21}^S$ and $\tilde{k}_{21}^N$ it is easy to calculate the initial relative price that would clear the world market for good 1. For the basic model, using equation (6.14) derived from the world market clearance condition for good 1, we can express the world relative price as

$$\tilde{p}_1^{Basic} = \left[ \frac{\Phi_1 l \left( N_1^N + N_1^S \right)}{\left( \tilde{N}_1^N \tilde{k}_1^N + \tilde{N}_1^S \tilde{k}_1^S \right)} \right]^{\alpha-\beta} \tag{6.58}$$

For the anticipation model, using the market clearance condition for good 1 and inserting the general expressions[34] for $\tilde{w}_1$, $\tilde{x}_{11}^N, \tilde{x}_{11}^S$, $\tilde{c}_{11}^N$ and $\tilde{c}_{11}^S$ into this equation, we can express the initial world relative price as

$$\tilde{p}_1^{Ant.} = \left[ \frac{\left( N_1^N \phi_{11}^N + N_1^S \phi_{11}^S \right) l}{N_1^N \tilde{k}_1^N + N_1^S \tilde{k}_1^S} \right]^{\alpha-\beta} \tag{6.59}$$

[34] Without the restriction of complete survival of adults into old age.

where $\tilde{k}_1^S = \tilde{k}_{11}^S + \tilde{k}_{21}^S$ and $\tilde{k}_1^N = \tilde{k}_{11}^N + \tilde{k}_{21}^N$, $\phi_{11}^i = \varepsilon\left(1-\left(1-\alpha\right)m_{11}^i\right) + \delta\left(1-\alpha\right)m_{11}^i$ with

$m_{11}^i = \dfrac{\theta\left(1+\rho\right)}{1+\rho+\lambda_1^i}$ for $i = N, S$.

For the basic model, the parameters of the utility and production functions and the initial population sizes of each generation in both countries are given in Table 6.1.

**Table 6.1: Parameters for the Numeric Solution of the Basic Model under Free Trade**

| **Country** | $\alpha$ | $\beta$ | $\theta$ | $\mu$ | $l$ | $A$ | $b_1$ [35] | $C_1, N_1, O_1$ |
|---|---|---|---|---|---|---|---|---|
| **S** | 0.3 | 0.5 | 0.4 | 0.8 | 1 | 10 | 0 | 1,1,1 |
| **N** | 0.3 | 0.5 | 0.4 | 0.8 | 1 | 10 | 0 | 1, 1, 1 |

As shown in the table, both countries are identical in terms of their consumer preferences, production technologies and initial population sizes. In simulation experiments, country $N$ and country $S$ will only differ in terms of their population dynamics. The first experiment, Trd.Sim.1, involves constant but different birth rates in $N$ and $S$. Each country is assumed to have an adult mortality rate of zero. As mentioned in Corollary 6, if adult mortality is zero, the choice of steady state birth rates under the basic model is constrained by the non-negativity constraint for the output of sector 1 in the low-fertility country $N$, i.e., $\dfrac{\varepsilon}{\Phi_1} \geq \dfrac{n^S}{n^N}$. The model parameterization given in Table 6.1 implies that $\varepsilon = 1.5090$ and $\Phi_1 = 1.3158$ so that

[35] This variable is the level of bequests received by each member of the initial adult generation.

$n^S$ and $n^N$ should satisfy $\frac{n^S}{n^N} \leq 1.1468$. Thus, fertility rates of $n^S = 1.1$ and $n^N = 1$ chosen for scenario Trd.Sim.1 satisfy the condition above so that the steady state labor and capital stocks used in sector 1 in country $N$ are greater than zero. As it is evident from Figure 6.1, the resulting population growth rates in $N$ and $S$ are 0% and 10% respectively.

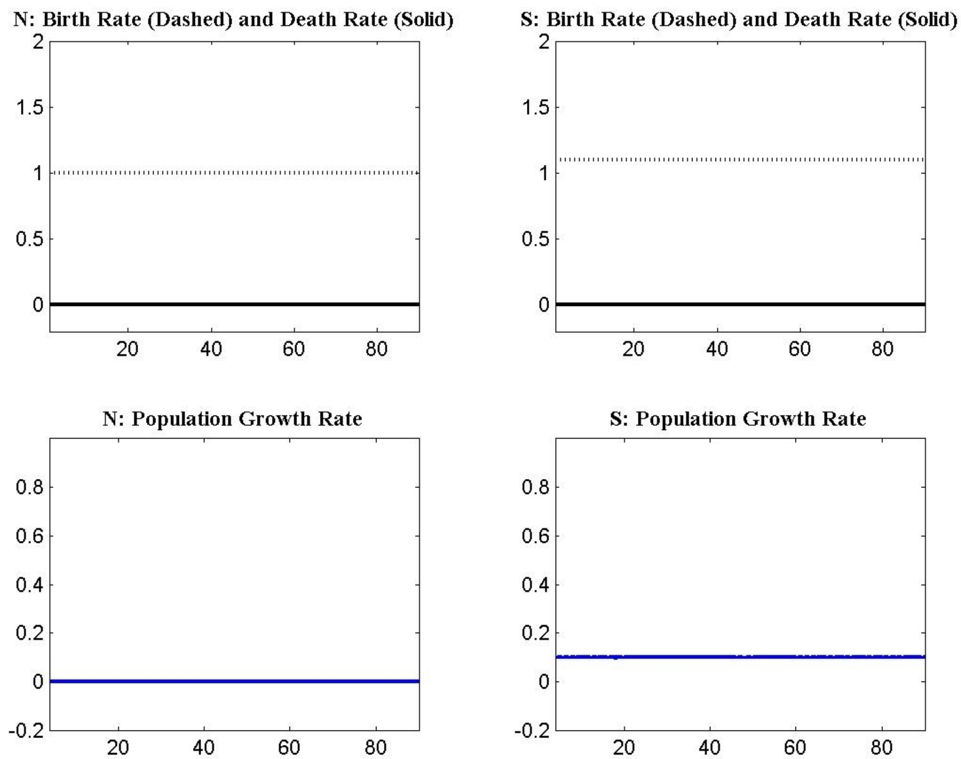


**Figure 6.1: Population Dynamics for Scenario Trd.Sim.1**

Figure 6.2 compares the autarky and free trade time paths of the important variables under this population scenario. The figure confirms the analytical results under the assumption that all adults survive into old age, which suggest that free trade lowers the relative price and the capital-labor ratio in the low-fertility country and worsens the welfare of the residents in this country. Indeed, the three left-hand side plots suggest that in the low-fertility country, there is a permanent reduction in relative price, capital per worker and welfare due to free trade. In the high-fertility country, on the other hand, there is a transitory improvement in these variables due to transition to free trade. It is evident from the right-hand side plots that the gains from free trade gradually diminish, as the economy gets closer to the steady state. Despite the welfare loss in country $N$ due to free trade, per worker capital stock and life-time utility levels in this country are still above those in country $S$, like in the case of autarky.

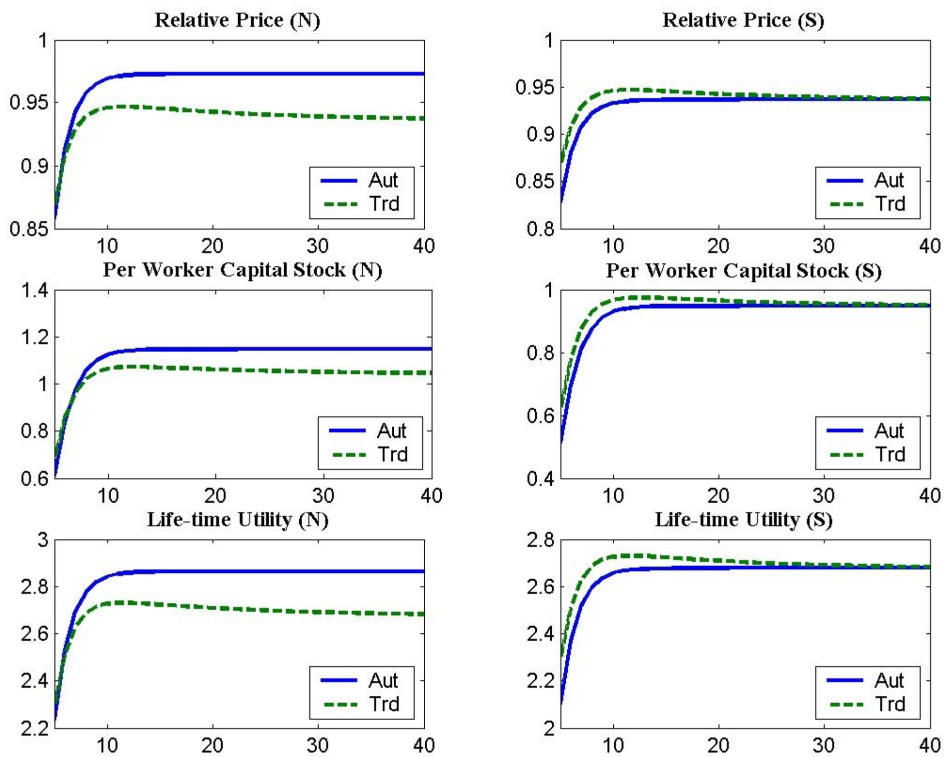


**Figure 6.2: The Results of Scenario Trd.Sim.1 for the Basic Model (**$n^N = 1,\ n^S = 1.1$**)**

The permanent gap between birth rates in $N$ and $S$ makes country $S$ relatively labor abundant and country $N$ relatively capital abundant. Thus, under free trade, country

$S$ gains a comparative advantage in producing the relatively labor-intensive child care good (good 1) and country $N$ gains a comparative advantage in producing the relatively capital-intensive consumption-investment good (good 2). Consequently, country $N$ ($S$) imports (exports) good 1 and exports (imports) good 2. Compared to autarky, free trade lowers (raises) the relative price of good 1 and real wages, and raises (lowers) the rental rates in country $N$ ($S$) towards the common world relative prices, wages and rental rates. This slows down the capital accumulation in the low-fertility country while speeding up the capital accumulation in the high-fertility country.

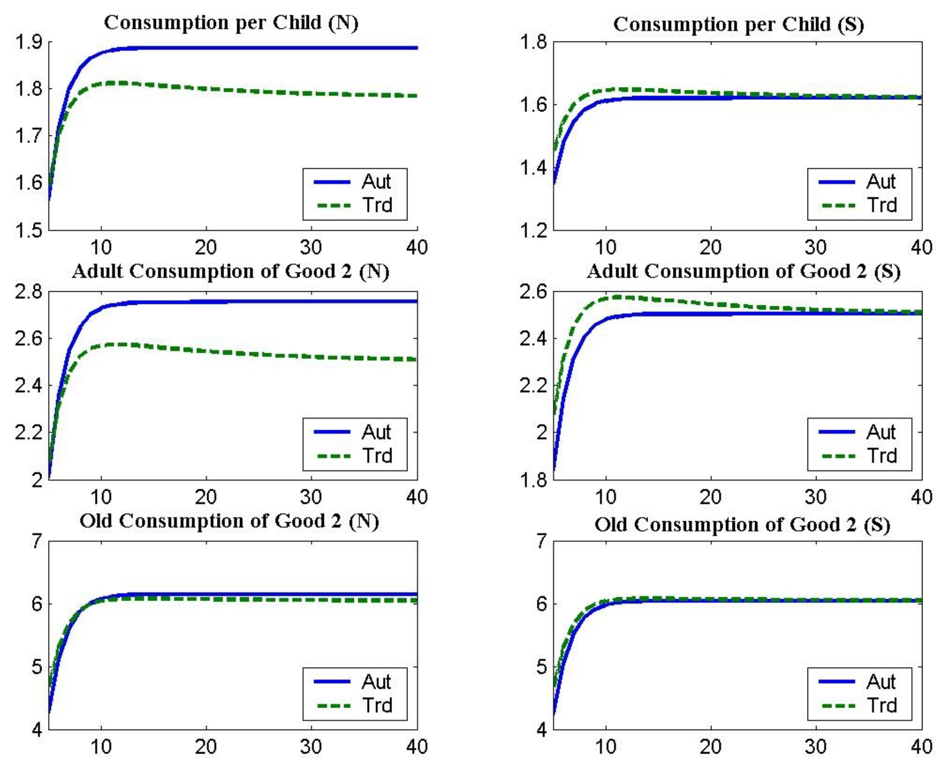


**Figure 6.3: The Results of Scenario Trd.Sim.1 for the Basic Model (Cont'd)**

Not surprisingly, the amounts of consumption of each generation in the low-fertility country are negatively affected by a transition to free trade, which lowers the life-time utility of all the future generations under the set of parameter values given in Table 6.1. In summary, the basic model verifies the finding in Sayan (2005) and Jelassi (2004) that differential population growth rates between two countries under a dynamic setting are by themselves sufficient to create a basis for trade, which paradoxically leads to a Pareto inferior outcome by making the low-fertility country worse off for the benefit of the high-fertility country.

Using the parameters and initial values given in Table 6.2, the same simulation experiment (Trd.Sim.1) with $n^N = 1$ and $n^S = 1.1$ is repeated for the anticipation model.

**Table 6.2: Parameters for the Numeric Solution of the Anticipation Model under Free Trade**

| **Country** | $\rho$ | $\alpha$ | $\beta$ | $\theta$ | $l$ | $A$ | $b_1$ | $C_1, N_1, O_1$ |
|---|---|---|---|---|---|---|---|---|
| **S** | 0.02 | 0.3 | 0.5 | 0.4 | 1 | 10 | 0 | 1, 1, 1 |
| **N** | 0.02 | 0.3 | 0.5 | 0.4 | 1 | 10 | 0 | 1, 1, 1 |

In Figure 6.4, the resulting common relative price, per worker capitals and utility levels under autarky and free trade are compared for each country. As the closed form solutions of the free trade and autarky models have already suggested, the anticipation model for free trade produces qualitatively identical results with the basic model since the survival probability of an adult into old age is one. Like in the basic model, as the gap between the adult populations in $S$ and $N$ widens, the high-fertility country $S$ starts to act as a large country and pulls down the world relative prices towards its autarky level of relative price.

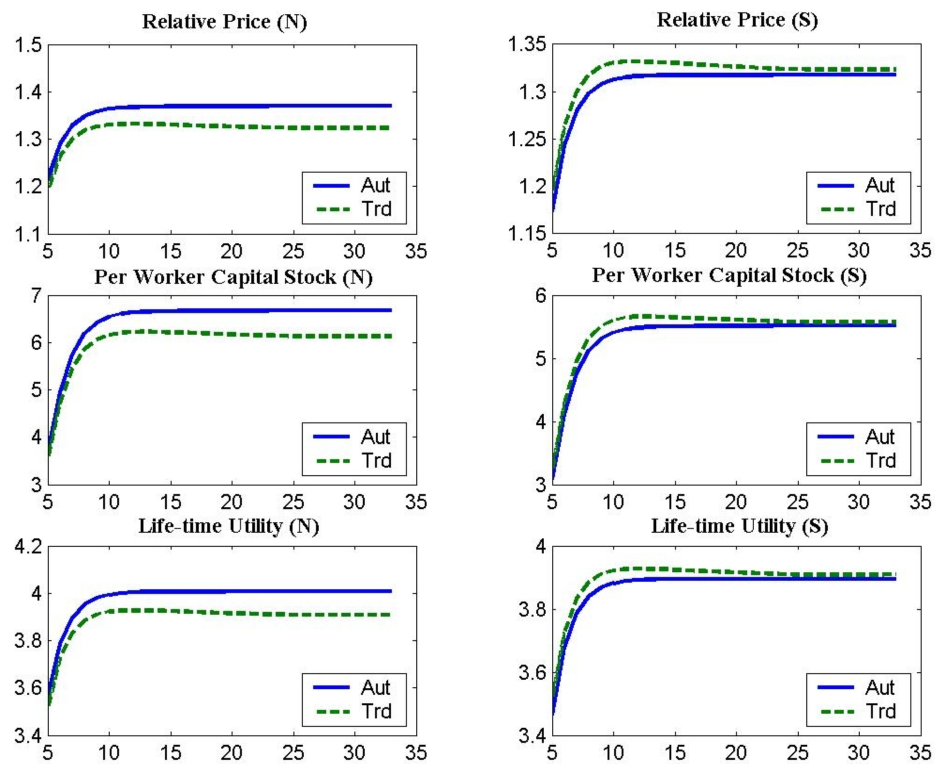


**Figure 6.4: The Results of Scenario Trd.Sim.1 for the Anticipation Model**

**($n^N = 1,\ n^S = 1.1$)**

To underline the distinctions between the behaviors of the basic and anticipation models under free trade, another scenario, Trd.Sim.2, is designed, where fertility and mortality rates are constant like in the previous experiment. As differently from Trd.Sim.1, in Trd.Sim.2 fertility rates are equal, but adult mortality rates are different across $N$ and $S$. It can be noticed from Figure 6.5 that the mortality rate in country $S$ is above that in country $N$. Under autarky, higher mortality in country $S$ implies lower old age dependency ratio and hence lower relative demand for good 2 compared to the autarky of $N$. This means a higher relative price and higher wage income in $S$ at the steady state. Thus, after transition to free trade, by the price equalization theorem, there will be a common world relative price which will be below the autarky price in $S$ and above the autarky price in $N$.

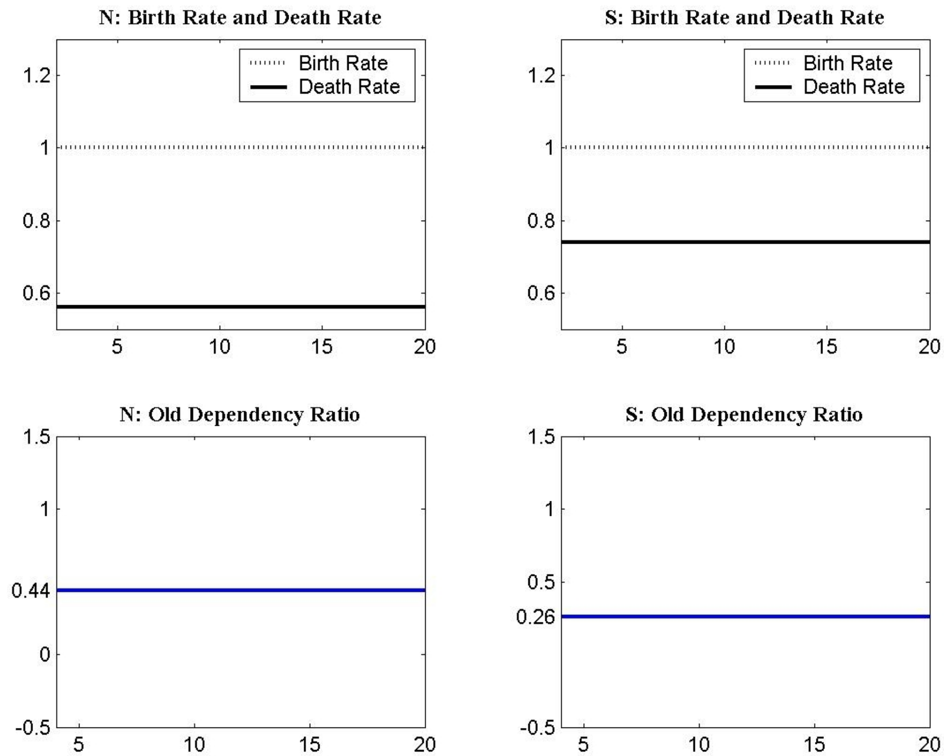


**Figure 6.5: Population Dynamics for Scenario Trd.Sim.2**

Since capital per worker is positively related to the relative price of the labor intensive good, free trade slows down the capital accumulation in high-mortality country and speeds up the accumulation in low-mortality country. Finally, the steady state welfare of high-mortality country –as measured by the life-time utility– is lower under free trade and vice versa in the other country implying that free trade is Pareto inferior to autarky in the long-run also when it is induced by permanent mortality differentials across countries. However, during the most of the transitional periods, relative price, capital per worker and lifetime utility in the high-mortality country is higher under free trade than under autarky (see Figure 6.6). This suggests that there are transition generations who gain from free trade, and hence, until a certain period of the transition, free trade may be Pareto improving over autarky. In other words, under the basic model, transition to free trade may be a socially desirable outcome for both countries, if this decision can given sufficiently early.

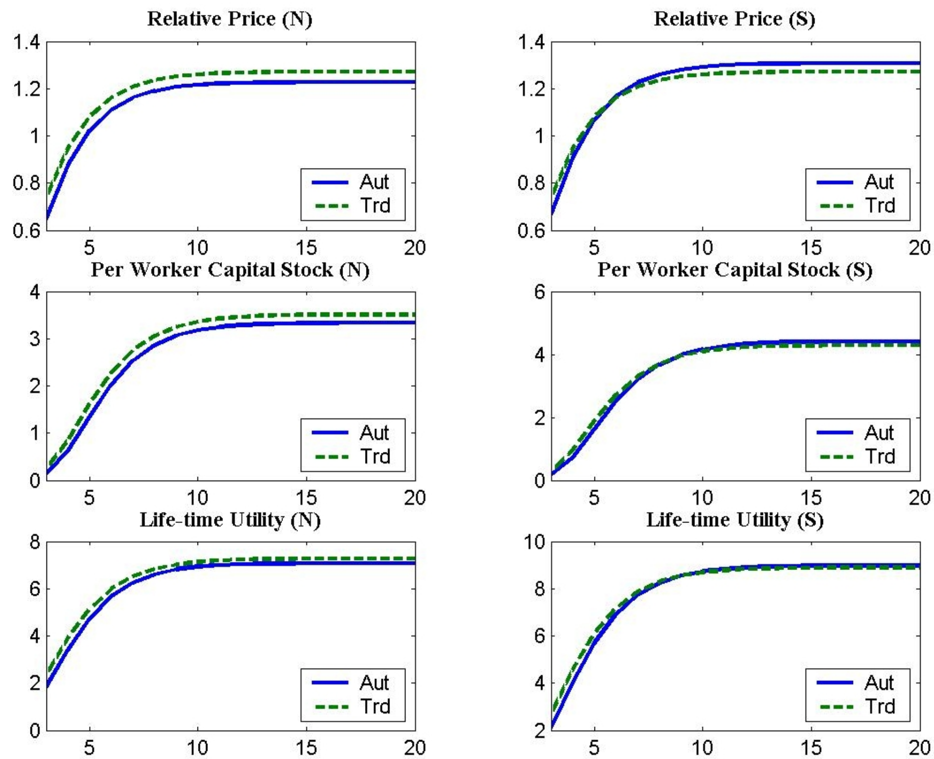


**Figure 6.6: The Results of Trd.Sim.2 for the Basic Model ($\lambda^N = 0.44$, $\lambda^S = 0.26$)**

Given that fertility rates in both countries are equal to one, the values of $\lambda^N = 0.44$ and $\lambda^S = 0.26$ are chosen to ensure that the old dependency ratios in $N$ and $S$ are consistent with the UN population projections for more developed and less developed (excluding least developed) regions in 2050[36]. Although adult mortality in $S$ is above

[36] In the model, old dependency ratio in country $i$ at the steady state is given by $E^i = \dfrac{\lambda^i}{n^i}$. Therefore $E^N = 44\%$ and $E^S = 26\%$.

that in $N$, population growth rates are equal to zero in both countries, just because the fertility rates are equal to one in both countries. The comparison between the autarky and free trade outcomes of this demographic scenario for the basic model is given in Figure 6.6.

Unlike Trd.Sim.1, where fertility rates differ across countries, in this scenario neither $S$ nor $N$ is able to act as a large country and determine the terms of trade at the steady state, because their populations grow at the same rate. This fact can also be observed from Figure 6.6, where the time path of world relative price neither converges to the steady state autarky price in $S$ nor to that in $N$.

Figure 6.7 shows the magnitude and direction of exports and imports of each good between two countries. Country $N$ with relatively low mortality exports (imports) the labor (capital) intensive good, and country $S$ with relatively high mortality exports (imports) the capital (labor) intensive good such that at each period the global good markets clear.[37] At the steady state, exports and imports of each good are positive implying that trade between the countries continues even after the steady state is reached. Trd.Sim.2 is a useful experiment because its results suggest that even in the lack of population growth differentials, a difference between life expectancies in two otherwise identical countries is sufficient for creating a basis for trade.

Figure 6.8 compares the autarky and free trade outcomes of Trd.Sim.2 under the anticipation model. Compared to autarky, the relative price, capital-labor ratio and expected utility of agents are lower in country $N$ and higher in country $S$ under free

[37] The negative values in the plots in Figure 6.7 indicate imports.

trade. Since the distance between the time paths of autarky and free trade scenarios is not clearly visible from Figure 6.8, the steady state values for the relative price and capital per worker from the numeric solutions of autarky and free trade scenarios for both types of models are reported in Table 6.3 for a better comparison.

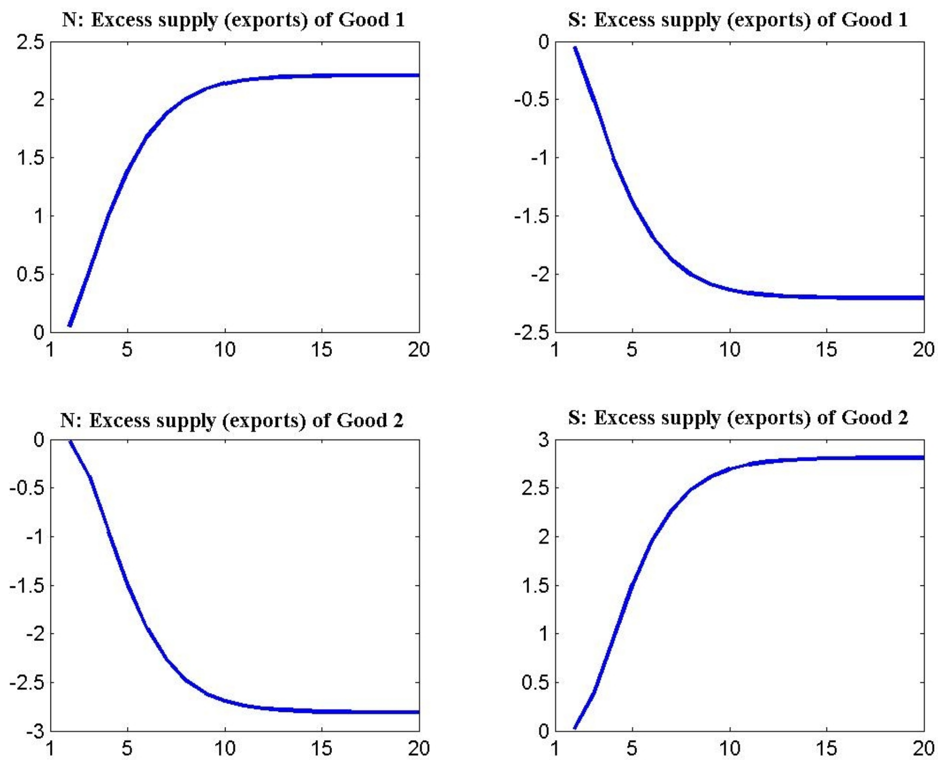


**Figure 6.7: Trade Flows between $N$ and $S$ under Scenario Trd.Sim.2 (Basic Model)**

Note that under the basic model, the autarky relative price and per worker capital in country $S$ (the high mortality country) are above those in country $N$ (the low mortality country), whereas under the anticipation model, the opposite is true.

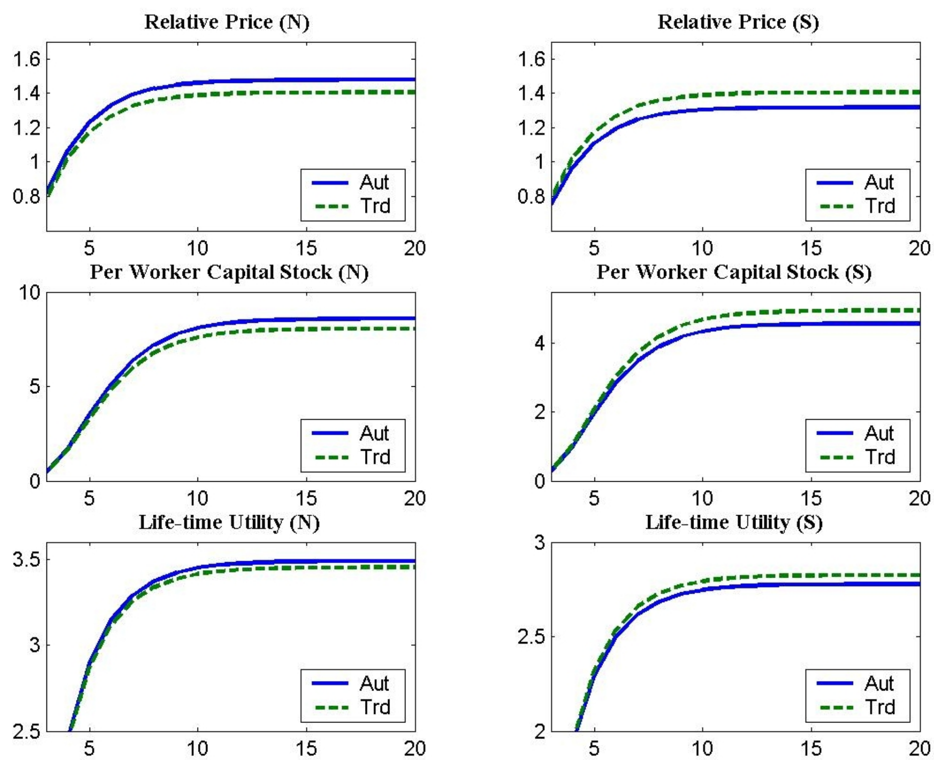


**Figure 6.8: The Results of Trd.Sim.2 for the Anticipation Model ( $\lambda^N = 0.44, \lambda^S = 0.26$ )**

Since the life expectancy in country $N$ is higher than in country $S$, country $N$ represents the more developed country with an older population, whereas country $S$ represents the less developed country with a younger population.

**Table 6.3: Comparison of the Steady State Values for $p_s$ and $k_s$ under Trd.Sim.2**

| Variables | Basic Model | | | | Anticipation Model | | | |
|---|---|---|---|---|---|---|---|---|
| | Country $N$ | | Country $S$ | | Country $N$ | | Country $S$ | |
| | Autarky | Trade | Autarky | Trade | Autarky | Trade | Autarky | Trade |
| $p_s$ | 1.2280 | 1.2719 | 1.3079 | 1.2719 | 1.4789 | 1.4051 | 1.3174 | 1.4051 |
| $k_s$ | 3.3366 | 3.5122 | 4.4171 | 4.2951 | 8.5927 | 8.0562 | 4.5806 | 4.9633 |

Therefore, the anticipation model more realistically captures the actually observed rankings of autarky capital-labor ratios in both countries than the basic model. In both models, the common world relative price settles between the autarky prices of the two countries due to free trade. However, the changes in relative prices and per worker capital stocks resulting from free trade are in opposite directions under the two models. Under the basic model, free trade lowers (raises) per worker capital and relative price of good 1 in the high-mortality country (low-mortality country). Under the anticipation model, however, free trade raises (lowers) per worker capital and relative price of good 1 in the high-mortality country (low-mortality country). In other words, the anticipation model predicts that adverse effects of high adult mortality are transmitted into the low-mortality country via free trade, while the positive effects of low mortality are transmitted into the high-mortality country. Despite this transmission effect, free trade level of per worker capital in country N remains above that in country S at the steady state (see Table 6.3). In line with these steady state results the direction of trade flows also differs across the two models. In contrast to the basic model where low-mortality country exports (imports) the labor-intensive (capital-intensive) good as shown in Figure 6.7, under the anticipation model the low-mortality country exports (imports) the capital-intensive (labor-intensive) good as shown in Figure 6.9.

Table 6.4 provides an overview of the comparative impacts of free trade on the steady state amounts of consumption and lifetime utility in both models. The results under the two models are in conflict with each other. The basic model predicts that in the low (high) mortality country, free trade raises (lowers) the amounts of consumption per child and adult, and lowers (raises) the old age consumption. The anticipation model predicts the opposite: Free trade lowers (raises) child and adult consumptions and raises (lowers) the old age consumption in low-mortality (high-mortality) country.

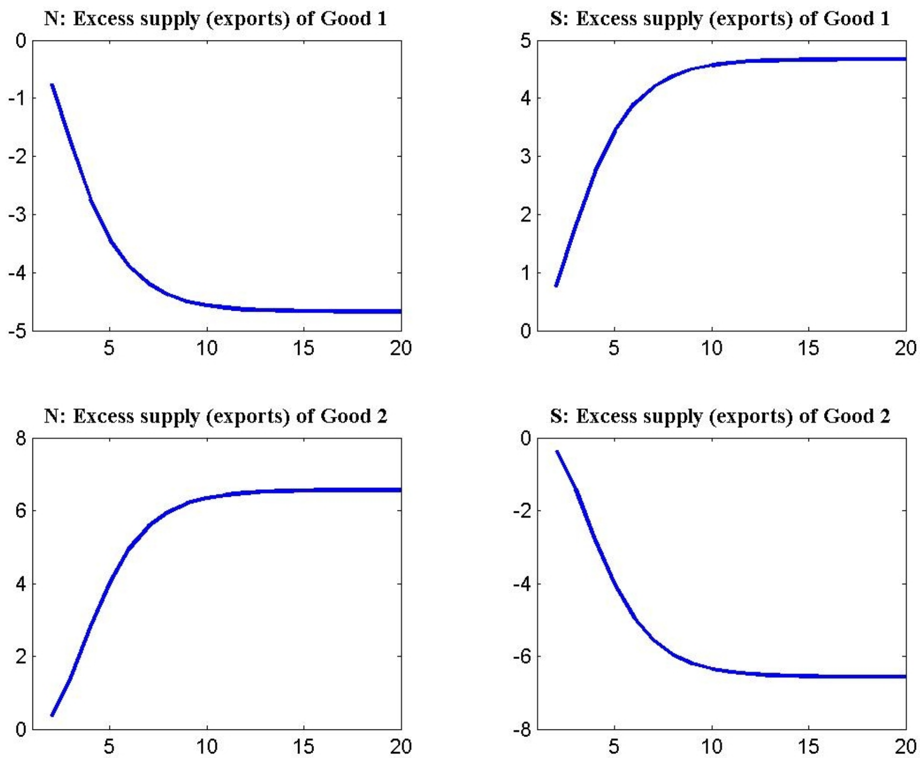


**Figure 6.9: Trade Flows between $N$ and $S$ under Scenario Trd.Sim.2 (Anticipation Model)**

One should be careful when comparing the basic and anticipation models in terms of the changes in utility levels. The utility levels reported for the basic model overestimate the realized life-time utility of the representative agent under positive adult mortality, because they represent the levels of utility that the agents expect to receive. In other words, since the adults cannot anticipate the real mortality rate and optimistically expect to survive into the next period with certainty, some of them actually cannot obtain the utility levels reported above. In the anticipation model, on the other hand, agents can perfectly foresee the mortality rate and hence the reported utility levels exactly correspond to their expected utility.

**Table 6.4: Comparison of the Steady State Consumption and Utility under Trd.Sim.2**

| Variables | Basic Model | | | | Anticipation Model | | | |
|---|---|---|---|---|---|---|---|---|
| | Country $N$ | | Country $S$ | | Country $N$ | | Country $S$ | |
| | Autarky | Trade | Autarky | Trade | Autarky | Trade | Autarky | Trade |
| $c_{1s}$ | 4.3475 | 4.4181 | 5.4034 | 5.4028 | 5.3876 | 5.3166 | 5.4562 | 5.5431 |
| $c_{2ys}$ | 8.0079 | 8.4294 | 10.6011 | 10.3081 | 11.9517 | 11.2055 | 10.7821 | 11.6829 |
| $c_{2os}$ | 11.4643 | 11.3474 | 13.6067 | 13.8765 | 21.7422 | 22.0681 | 13.9403 | 13.5958 |
| $U_s$ | 7.0761 | 7.2751 | 8.9820 | 8.8965 | 3.4905 | 3.4529 | 2.7770 | 2.8251 |

Keeping this distinction in mind, it should not be surprising to observe that free trade utility level in the anticipation model is higher for country $N$, despite the fact that the amounts of child and adult consumption in country $S$ are above those in country $N$. The reason for this is that the steady state amount old age consumption in $N$ is significantly higher than that in $S$ and the expected utility that an agent in country $N$ enjoys from old age consumption is higher than in country $S$ due to lower adult mortality in the former. Finally, it is clearly visible from Figure 6.8 that free trade is Pareto inferior to autarky because it worsens the welfare of all generations in the low-mortality country including the transition generations.

However, the welfare implications of the anticipation model are sensitive to demographic scenarios under free trade, as in the case of autarky. In other words, the impact of a transition to free trade regime depends on mortality rates in trading countries. Figure 6.10 compares lifetime utility gains and losses in each country due to free trade under two alternative demographic scenarios. Under both scenarios country $S$ is the high-mortality country and country $N$ is the low-mortality country. As evident from the figure, these scenarios produce opposite results: On the one hand, the country with relatively lower (higher) adult mortality gains (loses) from free trade, when life expectancies in trading countries are sufficiently high. On the other hand, the country with relatively higher (lower) adult mortality gains (loses) from free trade, when life expectancies in trading countries are sufficiently low.

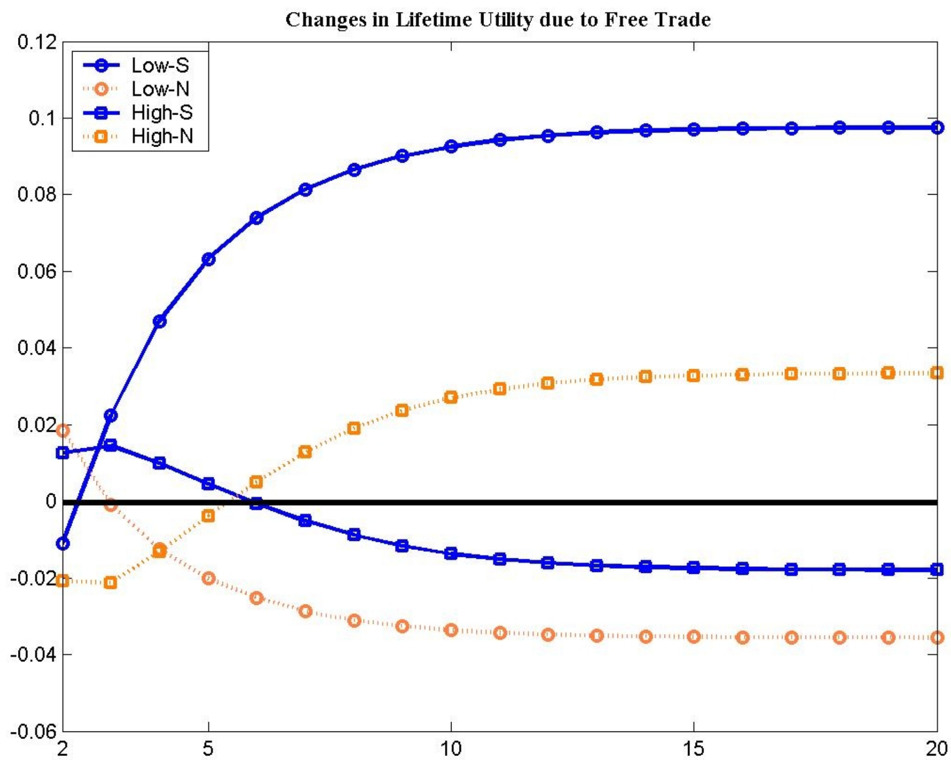


**Figure 6.10: Welfare Implications of Free Trade at Low vs. High Survival Rates (Anticipation Model)**

This result has interesting implications. It states that the welfare consequences of trade depend not only on the relative magnitudes of mortality rates in trading countries but also on the absolute magnitudes of mortality rates. This implicitly means that the timing of opening to trade is also crucial for whether free trade is welfare enhancing or welfare deteriorating. As a point in case, during early stages of demographic transition when survival rates are still low in both regions, trade between more and less developed regions is expected to limit the growth prospects in more developed regions with relatively higher longevity and to enhance the growth potential in less developed regions with relatively shorter longevity. At later stages of the demographic transitions in these regions when overall life expectancies reach a sufficiently high plateau, on the other hand, the more developed regions begin to reap the advantages of trade with less developed regions at the cost of the latter.

The next demographic scenario is referred to as Trd.Sim.3 and it involves a baby bulge in country $S$, whereas in country $N$ fertility rate is constant at the replacement level. In both countries the adult mortality rate is constant at 30% as shown in Figure 6.11. Since the mortality rate is constant in each country, the growth rates of the total populations in $N$ and $S$ are governed only by fertility rates. Figure 6.12 summarizes the effect of a baby bulge in country $S$ due to a temporary increase in fertility rates in Trd.Sim.3. The most remarkable role of free trade is that the negative effects of an increase or the positive effects of a decrease in fertility rate in one country under autarky is dampened under free trade, since the cost or benefit of the demographic shock is distributed between the trading parties due to the presence of a common world relative price. Figure 6.12 indicates that under free trade the reduction in relative price of good 1 in country  is indeed limited in magnitude compared to the autarky. Since the effect of a baby bulge under the anticipation model is similar to the basic model, the results of the anticipation model are not shown here.

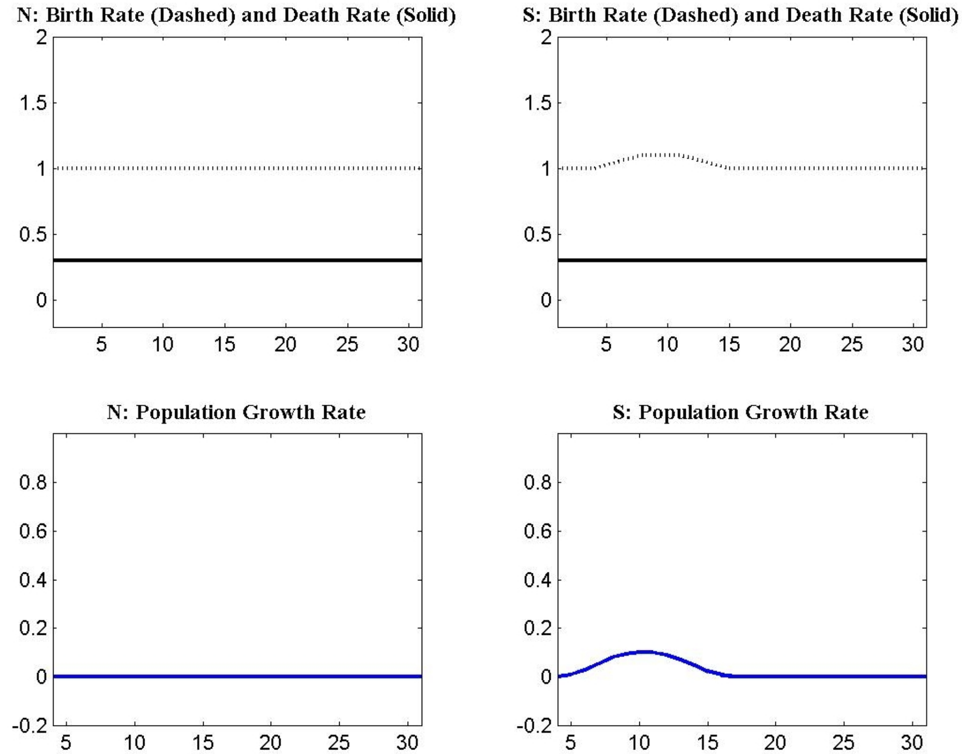


**Figure 6.11: Population Dynamics for Scenario Trd.Sim.3**

Analyzing the effects of a transitory shock to adult mortality under each model is a more interesting exercise to make a distinction between the basic and anticipation models. It is demonstrated in Chapter 5 that, as different from the basic model, under the anticipation model the direction of the impact of a rise in mortality can be negative for a certain range of population parameters. Hence, choosing a demographic scenario within this range of population parameters can also produce qualitatively different results under two types of free trade models. The population dynamics given in Figure 6.13 describes such a demographic scenario. Country $S$ experiences a positive and temporary shock to adult mortality. The mortality rate in country $S$ jumps from 40% to 60% and remains there for three periods before falling back to 40%. This shock results in a temporary decline in total population of country $S$ since the growth rate of population drops below zero. While mortality is constant at 60% during the following three periods, population growth rate remains constant at zero since fertility rate is constant at the replacement level of one.

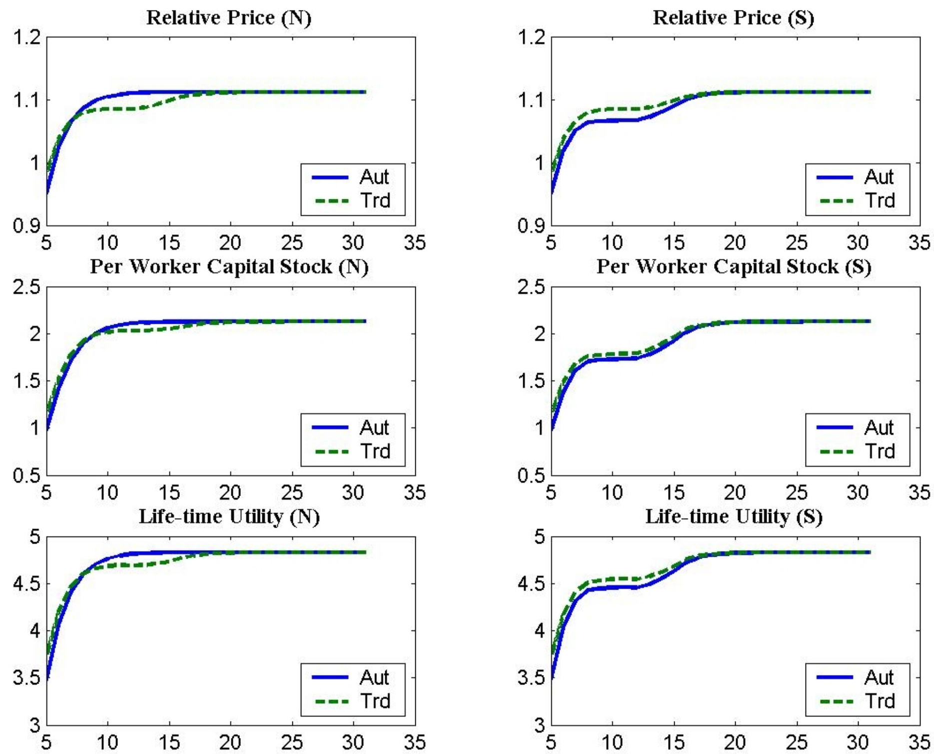


**Figure 6.12: The Effects of a Baby Bulge in Country $S$ under the Basic Model**

Figure 6.14 and Figure 6.15 show the impact of this shock on country and country under the basic and anticipation models respectively. The clear difference in the behavior of each model in response to this mortality shock reveals the critical role of mortality anticipation under free trade. Before the shock hits country $S$, the two countries have identical population dynamics, and hence there is no trade between them under the basic model. As the mortality rate starts to increase, the bequests received by each worker increase due to the scale effect and relative demand for good

2 declines causing a rise in the relative price of good 1 and a corresponding rise in real wages.

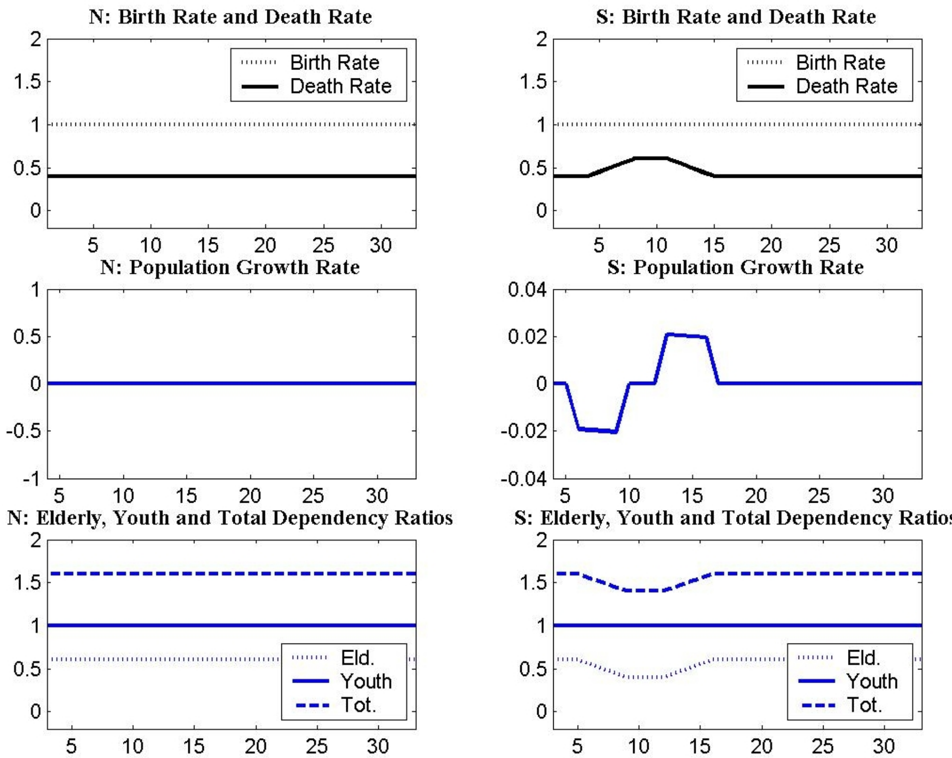


**Figure 6.13: A Transitory Increase in Adult Mortality in Country $S$**

As evident from Figure 6.14, a positive shock to adult mortality indeed raises the world relative price, but this rise is limited compared to the potential rise in the autarky price of country $S$. This price effect, however, remains limited relative to the case of autarky, because the decline in the global relative demand for good 2, i.e., the rise in global relative demand for good 1, is much more limited compared to the case of autarky. More precisely, as the world relative price tends to increase, a substitution in consumption from good 1 to good 2 at a global scale keeps the rise in the relative price of good 1 more limited than the rise under autarky of country $S$. The temporary increase in the relative price leads to a corresponding increase in the capital-labor ratio in country $S$. Compared to autarky during some periods of the transition to steady state, relative price and capital per worker under trade is higher in both countries. Then the direction of change is reversed in country $S$ and free trade relative price and capital per worker remain above the corresponding autarky levels until the steady state.

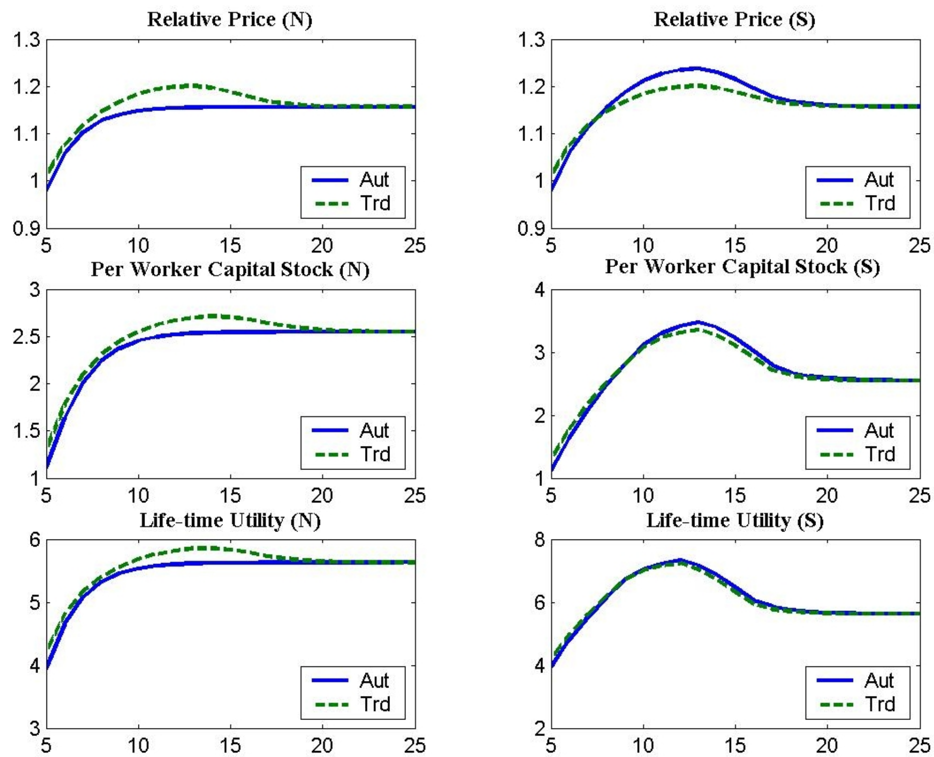


**Figure 6.14: The Effects of a Transitory Increase in Adult Mortality of Country $S$ (Basic Model)**

In country $S$, rising wage income due to price effect and the accompanying rise in bequests per worker both due to the positive income and increasing mortality effects result in higher savings per worker given the constant saving rate of $1-\mu$. This, in turn, leads to a rise in the capital-labor ratio in country $S$ as mandated by the capital market clearance condition. Country $N$, on the other hand, is only affected by the price change in the form of rising wage income, whereas a scale effect is missing here because, as differently from country, adult mortality of $N$ remains constant at 0.40.

Hence, the rise in bequests and savings is much more limited in $N$ than in $S$. This, in turn, explains why the upward swing of the capital-labor ratio in $N$ is smaller in magnitude compared to the swing in $S$ as seen from the middle row of Figure 6.14. Despite this, as the last row of the figure suggests, in the long-run, free trade improves the welfare in country $N$ and worsens the welfare in country $S$ when compared with the autarky scenario.

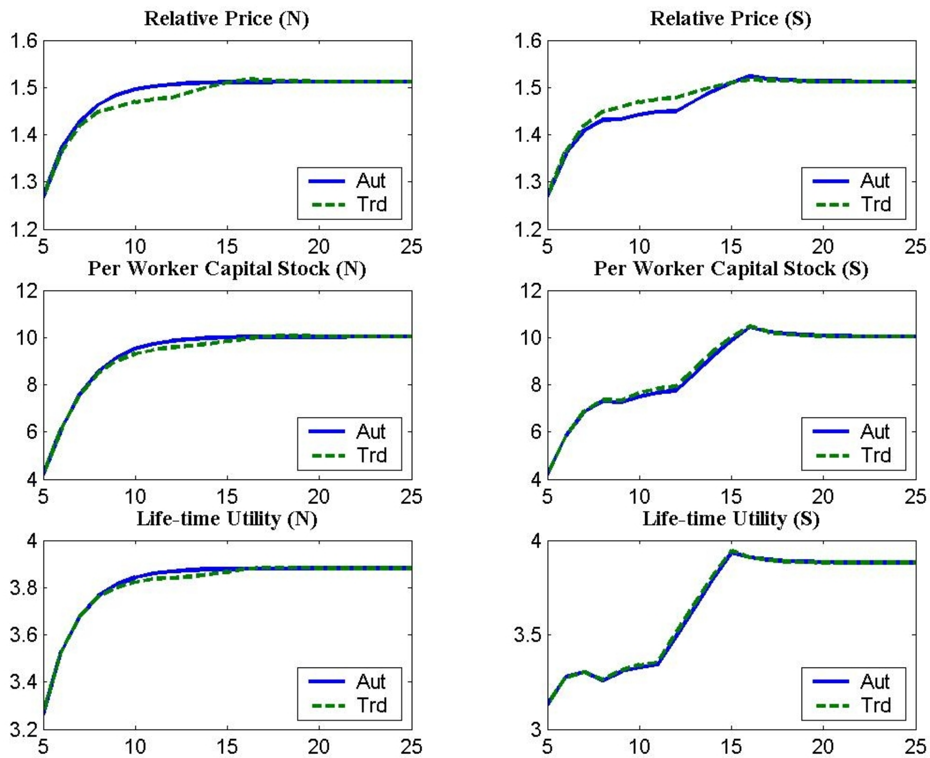


**Figure 6.15: The Effects of a Transitory Increase in Adult Mortality of Country $S$ (Anticipation Model)**

In contrast to the constant saving rate in the basic model, the saving rates in the anticipation model adjust to changes in mortality rate. The saving rate in country falls in response to increasing adult mortality and recovers after mortality falls back to its initial level, but the saving rate in country $N$ remains constant. Other things constant, including the fertility rate, the fall in saving rate leads to a capital dilution in $S$ since lower savings per worker mean lower capital per worker. As seen in Figure 6.15, country $S$ becomes capital scarce relative to the case without mortality shock. Again taking the 'no mortality shock' case as our benchmark, rental rates jump and wages fall making capital more expensive and labor cheaper.[38] Facing higher rental rates and lower wage rates, the producer of the labor-intensive good (good 1) increases its supply and the producer of the capital-intensive good (good 2) lowers its supply. At the same time, the relative price of good 1 falls relative to the benchmark without the shock.

Figure 6.16 reports the global effects of the temporary mortality shock on the consumption side. Although wage income of the representative agent in country $S$ is now below the benchmark level, due to a lower $p$ and higher propensity to consume in the adulthood –as a result of the lower saving rate, per child consumption of good 1 increases relative to the benchmark. Adult consumption is also higher, despite the negative price effect –implying that the inter-temporal substitution effect due to falling saving rate dominates the price effect. As expected, the olds' consumption is significantly lower than in the benchmark case. When the mortality starts to fall again towards its initial level, the system returns to the transition path of the benchmark case. As pointed out earlier, free trade partially absorbs the negative impact of the shock and smoothes out the downward swing in prices. As a consequence of the downward pressure on the world relative price, wage income and hence, savings in country  are also negatively affected by the shock. The transmission of the

[38] Note that since the shock hits country $S$ during the transition from a low initial capital stock to a higher steady state capital stock, the relative price and capital per worker show a monotonically increasing trend both under the autarky and free trade scenarios.

demographic shock from  to  makes country  worse off and country  better off as seen in Figure 6.15. This means that under certain demographic scenarios, the welfare implications of the same mortality shock may be totally different across the basic and anticipation models. For this particular demographic scenario discussed above, a temporary rise in adult mortality of one country creates a positive externality for the other country under the basic model, whereas under the anticipation model it creates a negative externality.

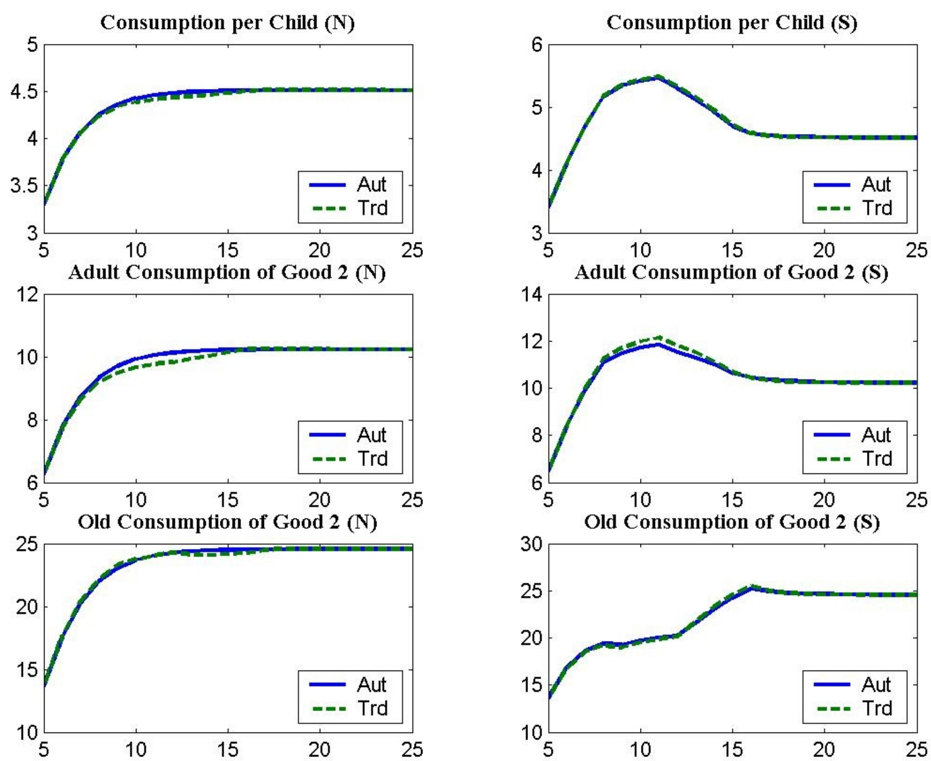


**Figure 6.16: The Effects of a Transitory Increase in Adult Mortality of Country $S$ (Anticipation Model) (Cont'd)**

## 6.4 Summary

This chapter has presented a thorough investigation of the economic implications of free trade in an environment where mortality and fertility rates are two independent parameters. As expected, under the restriction of complete survival of adults into old age, the implications of free trade are found to be almost identical to those reported in Sayan (2005) and Jelassi (2004). It is shown that under free trade, high fertility country gradually draws the world relative prices towards its autarky relative price by using the advantage of becoming a large country that is able to set the terms of trade. Among the two countries with different fertility rates, the one with higher fertility reaps temporary gains from trade whereas the one with low fertility becomes worse off due to free trade.

To fulfill the main purpose of the chapter numerical solutions of different free trade scenarios have been computed for both models under positive adult mortality. A summary of the demographic experiments and their qualitative results is given in Table 6.5. The results under Trd.Sim.1 reveal that regardless of the assumptions on mortality anticipation, trade under fertility differentials hurt low fertility country without changing the welfare of the high fertility country in the long-run. As for the effects of mortality differentials, the two models may produce exactly opposite results. Under the population parameters given in Trd.Sim.2, the basic model predicts a welfare gain for the low-mortality country and a welfare loss for the other. However, as demonstrated in Chapter 5, unlike the basic model where capital-labor ratio is increasing in mortality, the direction of the effect of a change in mortality varies with the baseline demographic scenarios upon which a particular shock is introduced under the anticipation model. This observation has naturally been influential in distinguishing the free trade impacts of demographic change under the two models. It is shown in Figure 6.10 that under the anticipation model and under certain population parameters, an increase in the mortality rate of country $S$ may be welfare improving (deteriorating) for $S$ ( $N$ ) under free trade compared to autarky.

**Table 6.5: Summary of the Results under the Main Demographic Scenarios in Chapter 6**

| | Trd.Sim.1 (Basic and Anticipation) | | Trd.Sim.2 (Basic) | | Trd.Sim.2 (Anticipation) | |
|---|---|---|---|---|---|---|
| | **Country S** | **Country N** | **Country S** | **Country N** | **Country S** | **Country N** |
| **Fertility Rate** | 1.1 | 1 | 1 | 1 | 1 | 1 |
| **Mortality Rate** | 0% | 0% | 74% | 56% | 74% | 56% |
| | **Direction of Change due to Free Trade** | | **Direction of Change due to Free Trade** | | **Direction of Change due to Free Trade** | |
| **Relative Price** | ↔ | ↓ | ↓ | ↑ | ↑ | ↓ |
| **Capital per Worker** | ↔ | ↓ | ↓ | ↑ | ↑ | ↓ |
| **Consumption per Child** | ↔ | ↓ | ↓ | ↑ | ↑ | ↓ |
| **Consumption per Adult** | ↔ | ↓ | ↓ | ↑ | ↑ | ↓ |
| **Consumption per Old** | ↔ | ↓ | ↑ | ↓ | ↓ | ↑ |
| **Lifetime Utility** | ↔ | ↓ | ↓ | ↑ | ↑ | ↓ |

Under the same set of population parameters, the same mortality shock produces the opposite result making country $S$ worse off and country $N$ better off compared to autarky. However, the direction of the welfare impact of trade under the anticipation model is sensitive to population dynamics. It is demonstrated that when the absolute levels of life expectancy in both countries are sufficiently low (high), free trade may make the low-mortality country worse off (better off) and the high-mortality country better off (worse off).

# CHAPTER 7:

# Conclusion

This thesis employs a 3x2x2 OLG GE framework to analyze the economic implications of demographic change under lifetime uncertainty using two models that differ with respect to the way expectations of mortality are modeled. In the basic model, agents behave as if there is full certainty about survival into the old age and thus have a constant saving rate. In the anticipation model, they can perfectly foresee their probability of survival into old age and adjust their saving rates accordingly. These models are solved both under autarky and free trade for various demographic scenarios.

Population projections presented in Chapter 2 suggest that the average life expectancy in more developed regions will remain above that in the rest of the world until the 2050s. In more developed countries where the effects of population aging are already visible, the average old age dependency ratio is 20 percent currently and is still increasing. Starting around 2015, less developed countries will also see significant increases in this ratio due both to declining fertility rates and increasing life span. Meanwhile, child dependency ratio in more developed regions will be more or less stable around 25 percent, whereas in less developed regions, this ratio will continue to decline mostly due to declining fertility rates.

These demographic trends are likely to create direct challenges for policy makers concerning the provision of health and education services, sustainability of social security systems and unemployment. Aside from these immediate effects, there will be indirect effects through changing relative factor abundances, relative prices and saving patterns in response to demographic change. This thesis concentrates on these indirect effects within a general equilibrium context.

As acknowledged in most of the recent studies, the age structure of population is a critical determinant of the growth potential of an economy. Hence, it is necessary to have a deeper and more extensive understanding of the separate and combined effects of fertility and mortality shifts, because these population parameters influence the age distribution in different ways. While the changes in fertility have a greater bearing on the child dependency ratio, old age dependency ratio is affected both by past fertility rates and current mortality rates among adults and elderly.

The first important question that is addressed in the thesis concerns the direction of changes resulting from mortality and fertility shocks under autarky. It is of particular interest to see the implications of the assumptions about mortality anticipation for the transitional dynamics of the two models. The second important question is about the free trade implications of fertility and adult mortality differentials across countries. Considering the increasing rate of globalization, this question is and will continue to be very relevant both for countries with relatively young and old populations that stand at different stages of demographic transition and economic development. More precisely, the thesis investigates whether free trade is Pareto improving or Pareto inferior under the presence of mortality differentials across trading countries? Since there is an ongoing debate about real extent of the relationship between private saving rates and life expectancy, whether the two different the basic model and the anticipation models vary in terms of their welfare implications under free trade is another interesting issue to tackle.

Motivated by these two main questions, this thesis aims to extend two different studies in different directions. The welfare implications of differential fertility rates under free trade have already been analyzed by Sayan (2005) using a 2x2x2 OLG model. This thesis extends it by adding the lifetime uncertainty component, i.e., adult mortality, and incorporating children as a third generation into the analysis. This thesis also extends the three-generation OLG model in Pecchenino and Utendorf (1999) developed to investigate the autarky implications of fertility and adult mortality dynamics in the existence of government and social security, by using a two-sector model and investigating the free trade implications of differential population dynamics. However, it excludes the government and social security from the analysis in order to concentrate on the first order effects. As differently from both studies, this thesis assumes that consumption goods of is generation-specific. Of the two goods considered, the labor-intensive commodity is viewed as an aggregate good composed of everything a child needs and hence is only consumed by children, whereas the capital-intensive commodity is consumed only by adults and the elderly while serving also as the investment good. These assumptions make the closed form solutions of the model easier to obtain under certainty of survival into old age, without changing the qualitative nature of the important results.

Assuming away adult mortality, i.e., assuming that agents survive to their last period with certainty, the analytical results are shown to be qualitatively identical across the basic and anticipation models. An increase in the fertility rate (or family size) lowers the relative price of the labor-intensive good, the capital per worker, i.e., the capital-labor ratio, and the wage rate, while raising the rental rate. This result is in line with previous empirical and theoretical studies arguing that higher fertility –or higher family size– slows down economic growth.

Under premature mortality characterized by probabilistic survival into old age, unintended bequests start to be influential on capital accumulation by acting as a channel of intergenerational wealth transfer. Unlike the changes in fertility rates, a shift in adult mortality affects the capital-labor ratio by directly altering the capital endowment instead of the labor endowment in the economy. The numerical results under positive adult mortality differ across the basic and anticipation models, because in the former, saving rates are constant, while in the latter, they adjust in response to changing mortality rates. Simulation experiments suggest that increasing adult mortality raises the steady state relative price of the labor-intensive commodity and the capital per worker, if agents optimistically assume that they will certainly survive into old age. On the other hand, if mortality rates are perfectly anticipated by agents, then the direction of the effects of mortality shifts on capital accumulation and the relative price is shown to depend on the initial population parameters. It is demonstrated through numerical experiments that under a given set of fertility and mortality rates, an increase in life expectancy raises (lowers) the capital-labor ratio, if the initial adult mortality and/or the initial fertility rate are/is sufficiently high (low). This finding has important policy implications for less and least developed countries, where fertility and mortality rates are relatively high. It suggests that the governments in such countries should spend a higher effort than more developed countries to improve the quality and the reach of the health services, in order to raise the private saving rates and help the economy to take off from its low initial capital stock.

The thesis also contributes to the literature by demonstrating that under lifetime uncertainty, the labor allocation between capital- and labor-intensive sectors is no longer neutral to demographic change. In particular, when adult mortality rate is positive, a decline in fertility or a rise in mortality increases (lowers) the share of effective labor employed in the labor-intensive (capital-intensive) sector. It is also shown that demographic change has important implications for intra-family welfare. Higher fertility increases the gap between child and adult consumptions, while reducing the amounts of both. As fertility rates decline, the intra-family inequalities in welfare start to diminish.

The thesis also adds to the dynamic trade literature. Assuming away lifetime uncertainty, the closed form solutions of both models under free trade support findings of Sayan (2005) and Jelassi (2004). It is demonstrated that under a dynamic Heckscher-Ohlin framework, free trade lowers the long-run capital-labor ratio and relative price in the low-fertility (older) country while not affecting it in the high-fertility (younger) country. In the long-run, the high-fertility country acts as a large country setting the terms of trade. The analytical results reveal that in the long-run, a rise in the fertility rate of the high-fertility country lowers the capital per worker in the low-mortality country. This indicates the presence of a demographic shock transmission channel from countries with a large population into those with a smaller population. The expected fall in fertility rates of developing countries may have welfare improving spillover effects on industrialized countries with lower fertility rates.

Other things constant, free trade does not change the long-run macroeconomic indicators in the high-fertility country, but it improves the capital-accumulation and welfare throughout the transition. In contrast to the case of autarky where decreasing fertility raises per worker capital stocks employed in both sectors, a decline in birth rates of the low-fertility country raises per worker capital employed in the capital-intensive sector and lowers that employed in the labor-intensive sector under trade. Therefore, in line with the real life observations, the models in the thesis predict that free trade tends to hurt labor-intensive sectors in industrialized countries with aging populations, as the world production of relatively labor-intensive goods such as agricultural goods shifts from industrialized to less industrialized countries. The intra-family distribution of welfare in industrialized countries is also affected by free trade. Other things constant, children and workers are those who are most negatively affected by the opening of free trade. Furthermore, if the lifetime uncertainty is included in the analysis, i.e., if the adult mortality rate is positive, any difference between fertility and/or survival rates is sufficient to make the amounts of consumption of each generation different across the trading countries. In this sense, the thesis improves the realism of the trade models described in Sayan (2005) and Jelassi (2004) by allowing the amounts consumption and hence, the welfare levels to become different across countries without relying on differences in consumer preferences but solely due to differential population dynamics.

As discussed in Chapter 3, the population dynamics in the model are such that population growth rates are governed only by fertility rates, if the life expectancy remains constant. Given this observation, perhaps the most important contribution of the thesis is the finding that a gap between life expectancies across countries creates grounds for free trade by creating a divergence in the capital-labor ratios and relative prices between these countries even when there is no difference between population growth rates. This result complements the findings of Sayan (2005) and Jelassi (2004) and other studies in the dynamic trade literature by adding mortality differentials to the list of factors that stimulate trade between two countries. Furthermore, differences in survival rates across countries do not by themselves lead to a divergence in population growth rates. Thus, if fertility rates are equal across two countries, neither of them can become a large country relative to the other and set the terms of trade. This, in turn, implies that unlike in the case of fertility differentials, in the case of mortality differentials, the gains and losses caused by free trade are permanent for each country. In other words, as demonstrated in Chapter 6, demographic shock transmissions may occur even among equally sized countries in the sense that none of the trade partners has the power to set the terms of trade.

Similar to the case of autarky, the basic and anticipation models may have opposite predictions about the economic implications of free trade when lifetime uncertainty is present. Under the basic model where saving rates do not respond to changes in mortality, the low-mortality country gains from free trade. Some generations born in the high-mortality country during the transition periods may gain from free trade despite that all the steady state generations are worse off under free trade. This implies that free trade may be Pareto improving over autarky in each country at least during the transition to steady state. This observation may have important implications regarding the decision of countries to start free trade or enhance the trade relations with other countries. In other words, transition to free trade or more extensive trade agreements may become socially desirable outcomes for both parties who can agree on trading with each other through democratic voting processes. Under perfect mortality anticipation, however, the numerical solutions of the free trade model under different population parameters suggest that free trade may raise as well as lower the capital-labor ratio and the overall welfare in the low-fertility country and vice versa in the high-mortality country. In particular, when the absolute magnitudes of mortality rates in trading countries are sufficiently high (low) but not identical, free trade tends to lower the welfare in the low-mortality (high-mortality) country and improve the welfare in the high-mortality (low-mortality) country. Secondly, the anticipation model does not permit a Pareto improvement under free trade throughout the transition as well as in the long-run. These two results distinguish the anticipation model from the basic model under free trade, and they emphasize the qualitative importance of the way the relationship between longevity and saving rates are modeled.

This thesis can be improved and extended in several directions. The first natural extension should be the incorporation of human capital formation as a labor augmenting force. In the current models, consumption spending of adults for the child support does not benefit children in the next period. In the extended model, parents would be investing on the education of their offsprings which, in turn, help children to build their human capital into adulthood. Under this extension, an increase in fertility rates is expected to further slow down economic growth due to decreasing human capital formation. A further extension in the same direction may involve endogenous technological progress driven by human capital formation. Alternatively, time devoted to research activity may determine the level of technology. Differential population dynamics may create a gap between the research efforts, and hence the technology levels, in more and less developed countries. Then, opening of trade may help the less developed country to close the initial technological gap. Finally, scientific discoveries and advancements in technology may be allowed to benefit from economies of scale in the sense that other things equal, larger populations may have a greater potential to make scientific discoveries from a probabilistic point of view. Adding such a feature into the human capital model would partially offset the negative impact of increasing fertility on human capital formation and technological progress.

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